Physics Factsheet

www.curriculumpress.co.uk Number 61

Answering questions using conservation of mechanical energy

The purpose of this Factsheet is to give guidance on how to approach AS and A2 questions involving conservation of mechanical energy.

Before studying the Factsheet, you should make sure that you are familiar with the concept of conservation of energy and all the important applications of mechanical energy such as: mass/spring system (Factsheet 53); simple pendulum (Factsheet 54), collisions and projectiles. You should also be familiar with the expressions for kinetic energy (K.E.) and gravitational potential energy (G.P.E.)

- You should know that collisions in which K.E. is conserved are called **elastic** collisions.
- It is important to appreciate that you are unlikely to get a question which concerns **only** conservation of mechanical energy. It is highly likely that forces, distances and speeds will also be involved.
- It is not always obvious that you need to use conservation of mechanical energy in order to answer a question. In questions asking about speed, it is often used through the relationship K.E. = ½ mv^2, so if a known amount of G.P.E has been changed into K.E (assuming no other transfer) then the speed of the object can be calculated.

Example 1. A ball of mass 150g is dropped from a height of 3m onto the ground. Calculate the speed of the ball when it hits the ground. Candidates often think that this question requires the use of equations of motion, but it is better approached from the viewpoint of conservation of energy.

Assuming no external energy transfers, G.P.E. lost = K.E gained
G.P.E lost = 0.15 × 9.81 × 3 = 4.4145J, this energy gives $\frac{1}{2}mv^2$

$$\text{So } \tfrac{1}{2} \times 0.15 \times v^2 = 4.4145$$

$$v^2 = \frac{4.4145 \times 2}{0.15} = 58.86$$

So v = 7.67m/s

In fact the mass was unnecessary, since this calculation could have been done in one step with the mass cancelling out.

$$\tfrac{1}{2} mv^2 = mgh \qquad v^2 = 2 \times g \times h$$

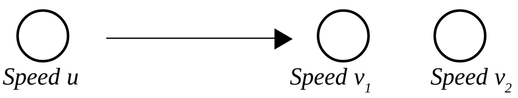
Example 2. Calculate the speed which a ball must be thrown vertically if it is to reach a height of 4m.

Assuming all the K.E is transferred to G.P.E, then
K.E. lost = G.P.E. gained
since at maximum height its speed is zero.

$$\text{So} \quad \tfrac{1}{2} mv^2 = mgh$$
$$v^2 = 2 \times 9.81 \times 4$$
$$v = 8.85 \text{ m/s}$$

- It is also highly likely that you will need to use the principle of conservation of momentum in the same question, since the use of conservation of momentum and conservation of K.E. in elastic collisions is a powerful tool in solving mechanics problems. (You should remember that momentum = $m \times v$)

Example 3 A ^{4}He nucleus travelling with speed u has a head-on collision with a particle of the same mass, which is at rest. If the collision is elastic, show that the incident particle comes to rest and the target particle moves off with the same speed that the incident particle had i.e. that the incident particle transfers all its K.E. to the target particle



Conservation of momentum gives:

$$mu = mv_1 + mv_2 \qquad (a)$$

Conservation of K.E. gives:

$$\tfrac{1}{2} mu^2 = \tfrac{1}{2} mv_1^2 + \tfrac{1}{2} mv_2^2 \qquad (b)$$

(a) gives $u = v_1 + v_2$ *(the m's cancel)*

Substituting for u in (b) with the ½ m's cancelled gives:

$$(v_1 + v_2)^2 = v_1^2 + v_2^2$$
$$v_1^2 + 2v_1v_2 + v_2^2 = v_1^2 + v_2^2$$

The only way that this equation can be satisfied is if $2v_1v_2$ *is zero, so either* v_1 *or* v_2 *must be zero.* v_2 *cannot be zero if* v_1 *is non-zero, because the incident particle would have to travel through the target particle, so the only realistic solution is that* v_1 *is zero. Thus* v_2 *must equal* v_i *i.e. the incident particle has transferred all its energy to the target particle.*

Note that this result is independent of the initial energy or mass.

Note also that momentum is a vector quantity, but here the velocities are all along the same straight-line, so vector addition is not necessary. In fact if the velocities are not all in the same straight line, the vector treatment of the equations above leads to the fact that the only way they can be satisfied is if the two particles move off at right angles to each other. This is observed in collisions between alpha particles in a cloud chamber filled with helium gas.

Example 4. A toy railway truck of mass 1kg, moves at a speed of 2ms^{-1} towards a second stationary truck of mass 2kg along a smooth track. (a) If the incident truck moves backwards with a speed of $^2/_3$ ms^{-1} after the collision, calculate the speed of the target truck after the collision. (b) Show that the collision is elastic.

(a) Let the target truck move off with speed v.
Conservation of momentum gives:

$$1 \times 2 = -1 \times \tfrac{2}{3} + 2v \qquad \text{(Remember velocity is a vector quantity, so moving backwards is written as minus)}$$

$$2v = 2 + \tfrac{2}{3} = \tfrac{8}{3}$$
$$v = \tfrac{4}{3}$$

The target truck moves forwards with speed $^4/_3$ ms^{-1}

(b) K.E. before the collision $= \tfrac{1}{2} \times 1 \times 4 = 2J$
K.E. after the collision $= \tfrac{1}{2} \times 1 \times \tfrac{4}{9} + \tfrac{1}{2} \times 2 \times \tfrac{16}{9} = 2J$

Since K.E. is conserved, the collision is elastic.

- You may also be asked questions in which the assumption that mechanical energy is conserved does not quite hold, and you may be asked to suggest why not.

Example 5: Passengers on a Ferris Wheel travel in a circle of radius 60m at a steady speed of about 12ms^{-1}.
(a) what is the change in the passengers' velocity between the bottom and the top of the wheel?
(b) A particular passenger has a change of G.P.E of 60KJ between bottom and top. What is her mass?
(c) The motor driving the wheel does not have to supply this energy, provided the ride is full. Explain why not.
(d) The motor does have to supply energy, though. Explain why.

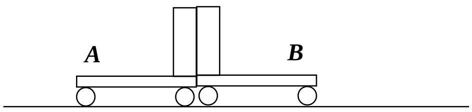
The first part of the question is to check your recollection that velocity is a vector quantity.
(a) If the Ferris wheel is moving anticlockwise, and taking left-to-right as the positive direction, then the velocity at the bottom is + 12ms^{-1} and that at the top is –12ms^{-1} so the change in velocity is 24ms^{-1}
(b) Change in G.P.E. = mgΔh , so $6 \times 10^4 = m \times 9.81 \times 120$

$$m = \frac{6 \times 10^4}{9.81 \times 120} = 60.0\text{kg (3.s.f)}$$

(c) Provided the ride is full, then for every passenger who gains G.P.E. another loses the same amount, so no energy needs to be supplied.
(d) Not all the energy is exchanged between G.P.E. and K.E., some is transformed into thermal energy in doing work against friction between the moving parts, some is transformed into thermal energy in doing work against air resistance and some does work in moving the structure itself.

- Questions often pose situations in which mechanical energy is **not** conserved, so it is important to recognize these as well.

The diagram shows two trolleys, initially at rest, in contact with each other on a smooth horizontal bench.
A spring-loaded piston is released which pushes the trolleys apart.



(a) State the total momentum of the trolleys as they move apart. Explain your answer.
(b) A has a mass of 0.8kg, B a mass of 1.2kg. If B moves off with a velocity of 1.5ms^{-1}, calculate the velocity of A.
(c) Calculate the total K.E. of the trolleys. Since the initial K.E. was zero, K.E. has not been conserved. Where has this K.E. come from?

(a) The total momentum of the trolleys as they move apart is zero. The initial momentum was zero. Momentum is conserved whenever there are no external forces acting. Being told that the bench is smooth entitles you to assume that no external forces are acting, though in practise this is not quite true, since there is still air resistance. Since momentum is a vector quantity, one trolley now has + momentum and one –, thus this can add up to zero.

(b) $M_BV_B + M_AV_A = 0$
$1.2 \times 1.5 + 0.8 \times V_A = 0, \quad 0.8 \times V_A = -1.8$
$V_A = -2.25\text{ms}^{-1}$
The minus sign is important, since it indicates that trolley A moves backwards.

(c) $K.E_A = \frac{1}{2} M_AV_A^2 = 0.5 \times 0.8 \times 2.25^2 = 2.025J$
$K.E_B = \frac{1}{2} M_BV_B^2 = 0.5 \times 1.2 \times 1.5^2 = \underline{1.35J}$
Total K.E. = $\underline{3.375J}$
This K.E. has been transformed from elastic energy which was stored in the spring

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

A boy, of mass 45kg, is sliding on ice at a constant 3ms^{-1}. When he collides with another boy of mass 50kg who is travelling at 2ms^{-1}, they become entangled and move off together.

(a) Calculate the speed with which they move off, stating any assumption you make. (3)
Momentum is conserved.
$M_1V_1 + M_2V_2 = M_tV_t$
$(45 \times 3) + (50 \times 2) = 95 \times V_t$
$135 + 100 = 95 \times V_t$
$V_t = 2.47\text{ms}^{-1}$ *2/3*

> The candidate has scored 2 marks for a correct calculation, but conservation of momentum is not an assumption. The assumption is that there are no external forces acting, in which case momentum is conserved.

(b) Discuss the extent to which the assumption is true. (2)
The principle of Conservation of Momentum states that momentum is always conserved. 0/2

> The candidate has missed the point of the question, which was to test understanding of the conditions in which the Principle of Conservation of Momentum can be used.

(c) This collision is not elastic, calculate the change in K.E.(3)
K.E before = $(\frac{1}{2} \times 45 \times 9) + (\frac{1}{2} \times 50 \times 4) = 302.5J$
K.E after = $\frac{1}{2} \times 55 \times 2.47^2 = 167.8J$
Change in energy = 134.7 J
The K.E. has decreased by 134.7J 3/3

> The candidate has performed the calculation correctly.

(d) What has happened to the apparently "lost" energy? (2)
It has been dissipated as thermal energy. 0/2

> A lot of candidates' first, unthinking, response to energy "loss" is to quote energy dissipated as thermal energy. Here the assumption has been made in calculating the final speed that momentum is conserved. That requires no external forces acting, hence no friction, so no energy is dissipated as thermal energy.

Examiner's Answers
(a) assumption is that there are no external forces acting, in which case momentum is conserved.
$M_1V_1 + M_2V_2 = M_tV_t$
$(45 \times 3) + (50 \times 2) = 95 \times V_t$
$135 + 100 = 95 \times V_t \qquad V_t = 2.47\text{ms}^{-1}$
(b) The assumption that there are no external forces acting is a reasonable approximation on ice, where the frictional force is very low.
(c) K.E before = $(\frac{1}{2} \times 45 \times 9) + (\frac{1}{2} \times 50 \times 4) = 302.5J$
K.E after = $\frac{1}{2} \times 55 \times 2.47^2 = 167.8J$
Change in energy = 134.7 J
The K.E. has decreased by 134.7J
(d) The internal energy of the boys has been altered in the collision.

Acknowledgements: *This Physics Factsheet was researched and written by* ***Janice Jones.***
The Curriculum Press,Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU.

ISSN 1351-5136