'A' LEVEL PHYSICS - REVISION

Suggested method: Read notes and text books. Have paper to hand and jot down points of relevance. Learn by heart definitions and formulae that require this treatment.

Study the methods of experiments described in the course, demonstration experiments and experiments done by you in the lab. Note points of detail as well as the main principles and associated mathematics.

If there is anything you do not understand - ASK. MOST IMPORTANT - having completed your study go to as many questions as possible in past exam. papers and your text book and then try to answer them without further reference. If you cannot do them, you have not done enough work.

It is probably a good idea to make your own set of KEY FACTS as a summary.

I will set you a test or problems each week from now on in the following sections:-

1. Particle dynamics. Newtons Laws, Units and dimensions. Vectors. Impulse. Momentum.

2. Angular momentum. Kinetic and potential energies. Conservation of energy and momentum.

3. Circular motion. S.H.M. Oscillations - free, damped and forced. Resonance.

4. Gravitation. Keplar and Newton.

5. Elasticity. Young's modulus. Strain energy. Crystal structure.

6. Waves - travelling and stationary. Velocity wavelength frequency and amplitude. Reflection, refraction. Superposition. Interference. Beats.

7. Diffraction. Young's slits. Diffraction gratings. E.M. radiation, wave nature, polarisation. Telescopes and resolving power.

8. Temperature. Thermometers. Thermal expansion. Conductivity.

9. Ideal gases. Kinetic theory of gases. Real gases. S.V.P.

10. Thermodynamics. Zeroth and First Law. Isothermal and adiabatic changes.

The above headings are not complete. Work in conjunction with your syllabus and go beyond it rather than the other way. Work conscientiously and keep up to date. You can always gain a deeper understanding even if you think you know the work already.

A.W.R. January 1979.

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Dimension Problems.

Solving and/or checking of formulae via dimensioning.
Both sides must be dimensionally equivalent.

Example. Dimensions:

Energy in terms of velocity.	MLT^{-1}
mass	M
Acceleration	LT^{-2}

Energy dimensions ML^2T^{-2}

$$\therefore ML^2T^{-2} = K M^A (LT^{-1})^B (LT^{-2})^C$$

Equating coefficients:

M: $1 = A$

L $2 = B + C$

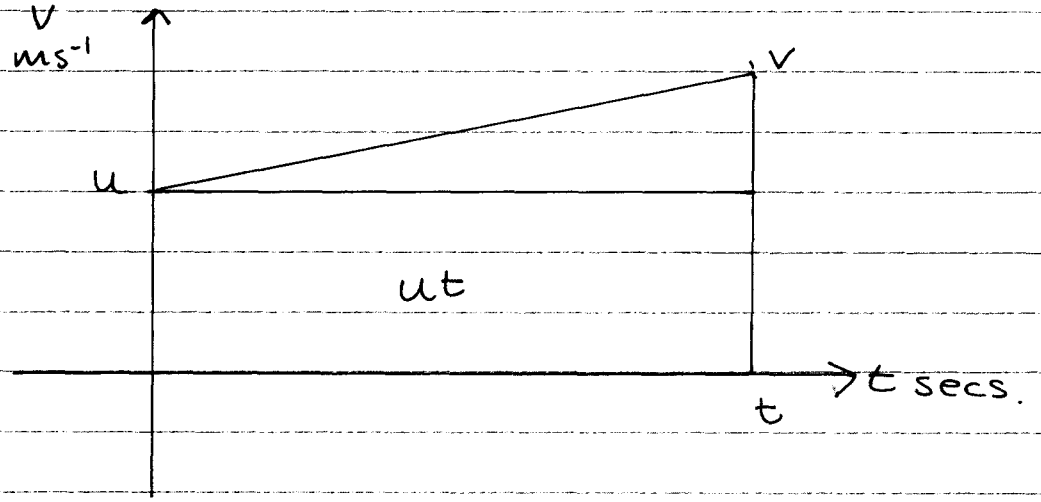
T $-2 = -B - 2C$

$a = b = 2 \quad c = 0$

$E = KMV^2$ by expt $K = 1/2$

$$\therefore E = \frac{1}{2}MV^2$$

Proof of $s = ut + \frac{1}{2}at^2$



S = Area under curve.

$$S = \int_0^t v\,dt = ut + \frac{1}{2} \times t \times (v-u)$$

but $a = \frac{v-u}{t}$ $\therefore$ $\frac{1}{2}t(v-u) = \frac{1}{2}at^2$

$\therefore$ $\underline{s = ut + \frac{1}{2}at^2}$

Momentum
+ Newtons Laws.

Momentum is the inertia of a body

Conservation of momentum, states that in any system of interacting bodies total momentum remains constant given no external forces.

Collisions.

(I) Inelastic - Particles Adhere or Bounce of inelastically Momentum conserved in collision but ~~kinetic~~ energy is not.

(II) Elastic
Momentum + Energy conserved so solve using quadratic equations.

Impulse. (Newton seconds)

$$\text{Impulse} = \int_a^b F\,dt$$

Impulse is a force acting over a certain period of time. = Change of momentum.

Momentum + Change.

Momentum = mv = Ft (Impulse)

Change of momentum = $\frac{mu - mv}{t}$ = ma.

Newtons Laws of Motion.

1. A body continues in a state of rest or in uniform motion in a straight line unless acted upon by a force.

2. The rate of change of momentum is directly proportional to the resultant applied force taking place in the direction of the resultant force.

3. For every action there is an equal and opposite reaction.

Lower Sixth

Physics Problems - Momentum

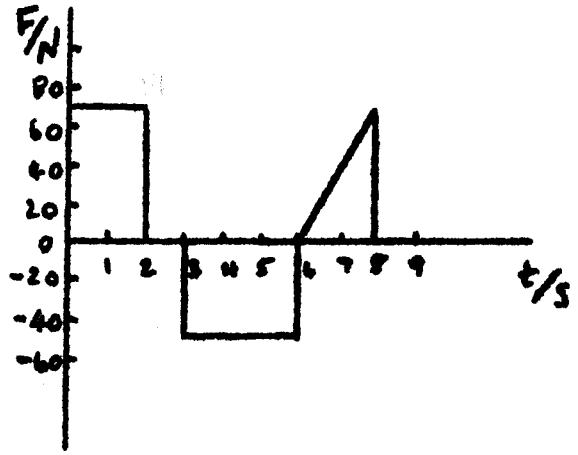
"If you bump into difficulties, keep on going!"

1) A stationary billiard ball of mass 0.20 kg is hit by a cue which exerts an average force of 60 N over a time of 8.0×10^{-3} s. Calculate the speed and k.e. of the ball after impact

2) The forces shown on the force-time graph are applied to a body of mass 10 kg

a) Calculate the total impulse after 8.0 s

b) Assuming that the body started from rest, draw (i) an acceleration-time graph (ii) a velocity-time graph (iii) and a momentum-time graph.

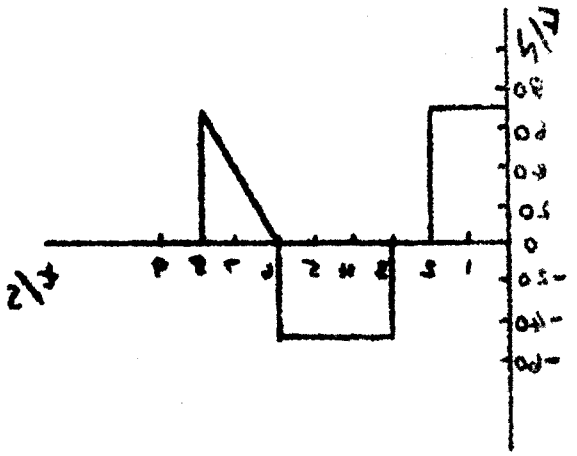


F=ma

3) A ball of mass 2.0 kg, travelling at 1.5 $m s^{-1}$, catches up and collides head on with another ball of mass 3.0 kg travelling at 0.80 $m s^{-1}$ in the same direction. If the collision is perfectly elastic, calculate for each ball a) the velocity after collision and b) the change in momentum.

4) A nucleus, which is initially at rest, emits a positron of momentum 9.2×10^{-23} Ns and a neutrino of momentum 5.3×10^{-23} Ns. The angle between the directions of travel of the two particles is 90°. Calculate the values of the following for the new nucleus, whose mass is 3.9×10^{-25} kg: a) the magnitude and direction of its momentum b) its velocity and c) its kinetic energy

AWR.



Physics Problems - Dynamics

How do these strike you?

(Take "g" to be $9.8\ m\,s^{-2}$)

1, Draw free body diagrams for the body underlined in each of the following situations, and describe the forces a) a BOOK resting on a table

b) an ELECTRIC LIGHT BULB hanging by flex from the ceiling

c) a SLEDGE being pulled by a rope attached to a dog

d) a PEN lying on a TABLE which rests on the floor

e) a SUITCASE being dragged off a TABLE by a MAN who is standing on the floor

f) a STONE being fired from a CATAPULT, while the stone is still in contact with the catapult.

2, A lift is rising with an acceleration of $0.5\ m\,s^{-2}$ What is the push of the lift on a man of mass 60kg who is standing on the floor of the lift.

3, A plumb-line hangs from the roof of a train which is moving on a horizontal track. If the plumb-line makes an angle of 3° with the vertical, what is the acceleration of the train?

4, A massless rope passes over a smooth massless pulley and is attached at its ends to masses of 4.0 kg and 6.0 kg. What is the acceleration of the 4.0 kg mass and what is the tension in the rope?

5, A locomotive whose mass is 1.0×10^5 kg pulls two coaches each of mass 5.0×10^4 kg. The ground exerts a forward force on the locomotive of 1.0×10^5 N and the resistive forces total 3.0×10^4 N on the locomotive and 1.0×10^4 N on each of the coaches. Find the acceleration of the train and the pull which the locomotive exerts on the first coach, and the pull which the first coach exerts on the second coach.

Andrew

Dynamics.

Involves Work / Energy / Power.

Work is said to be done when the point of application moves and is done in that direction.

Energy is the capacity to do work.

Power is the rate of doing work.

Work done = Force x Distance x Phase Angle
= $F \times \cos\theta$ (Dot product).

$$\text{Power} = \frac{\text{Force} \times \text{Distance}}{\text{time}}$$

$$= \frac{Fd}{t} = Fv \text{ or } F\frac{ds}{dt}.$$

Energy:

Rotational. $= \frac{1}{2} I_0 \omega^2$

I_0 = moment of inertia $= \Sigma \delta m x^2$
ω = angular rotation.

kinetic energy $= \frac{1}{2} m v^2$

M = mass body
V = |velocity|

Potential energy $= mgh$.

g = Acceleration due to Gravity.
h = Distance above some zero reference point

Units of Energy are Joules.

Gravitation.

Theory.

Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distances apart.

Newtons Law $F = G \cdot \frac{m_1 m_2}{r^2}$

G = Gravitational constant = $6.673 \times 10^{-11} Nm^2kg^{-2}$

m_1, m_2 are the masses of the 2 bodies.

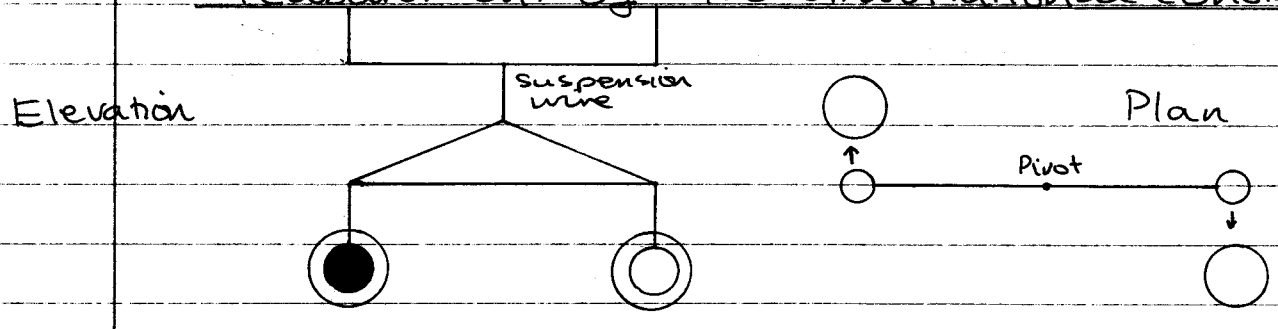
r distance between masses.

Coulombs Law $F = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{r^2}$

$Q_1 Q_2$ are the charges on the bodies.

r distance between charges.

Measurement of the Gravitational constant G.



Small sphere large sphere behind

Large sphere infront small sphere behind.

The apparatus is enclosed to reduce the effect of air.

System will be by θ radians.

At Balance: Gravitational torque = elastic restoring torque.

The suspension wire produces the elastic restoring torque.

m = mass Little sphere.
M = mass of Large sphere.
C = suspension constant.
x = Distance between spheres. larget small.
l = Distance between small spheres.

$$G\frac{mMl}{x^2} = C\theta$$

System deflected by θ rads.

The apparatus has been improved by Boys to incorporate following features of a quartz fibre suspension:

1. great tensile strength.
2. small radius and shear modulus (low C)
3. Size of apparatus reduced considerably.

MVI Physics Revision Test [2]

1, A 4 kg ball moving with a velocity of $10.0\ m\,s^{-1}$ collides with a 16 kg ball moving with a velocity of $4.0\ m\,s^{-1}$ (i) in the same direction (ii) in the opposite direction. Calculate the velocity of the balls in each case if they coalesce on impact and the loss of energy resulting from the impact. State the principle used to calculate the velocity.

2, A bullet of mass 4 g enters a block of wood of mass 996 g suspended by a string 6 m long. The block moves sideways from its mean position by 2 m before coming to rest.

a) What is the PE of the block in this position?

b, What was its KE immediately after impact?

c, What was its momentum immediately after impact

d, Hence what was the velocity of the bullet on impact.

3, Calculate the percentage loss of kinetic energy of a neutron of mass m when it makes a perfectly elastic head on collision with a stationary beryllium nucleus of mass 9m.

AMR

Field Treatment of Space Travel

Potential of a body is the potential per unit mass

E.g. Consider a satellite of mass m
Distance r from the earths centre
E_p = potential energy.

$$E_p = -G\frac{mM}{r}$$

If the satellite is moving in a circular orbit with velocity V ms^{-1}

Total energy = Sum Potential + kinetic

$$E_{Total} = E_p + E_k = -G\frac{mM}{r} + \frac{1}{2}mv^2$$

but we can express E_k in terms of G, m, M, r by using the centripedal force.

$$F = G\frac{mM}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}G\frac{mM}{r}$$

$$E_T = -G\frac{mM}{r} + \frac{1}{2}\frac{mM}{r}\cdot G = -\frac{1}{2}G\frac{mM}{r}$$

Total Energy $E = -\frac{1}{2} G \frac{mM}{r}$

If R is the Radius of the earth then a body of mass m has energy

$$-G\frac{mM}{R}$$

Extra energy to put satellite into orbit is

$$E_L \quad -\frac{1}{2} G \frac{mM}{r} - \left(-\frac{GmM}{R}\right) = GmM\left(\frac{1}{R} - \frac{1}{2r}\right)$$

If orbit is close to earths surface then $r \approx R$.

$$\therefore E_L = GmM\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{1}{2}\frac{GmM}{R}$$

Energy required at Lift off $= \frac{1}{2} G \frac{mM}{R}$ Joules.

If this energy is supplied at lift off and u is the escape velocity then:

$$\frac{1}{2}mu^2 = \frac{1}{2}\frac{GMm}{R} \Rightarrow u^2 = \frac{GM}{R}$$

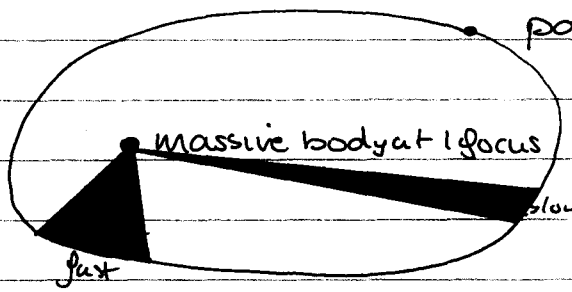
Escape velocity $= \frac{GM}{R}$

Keplers Laws.

Kepler a mathematician of 1618 put forward 3 laws.

1. **Law of Orbits**
The path of a small body is a conic section having the large body as one focus. The particular section is determined by the speed of the small body at a given instant.

2. **The Law of Areas**



Equal areas in equal times.

The area swept out in a given time by the radius vector joining the large body to the small is always the same.

3. **The Law of Periods.**
For all planets the time period T is related to the semi major axis R by the relation $T^2 \propto R^3$

$$\text{i.e } \frac{\text{Orbital Period}^2}{\text{Semi major axis}^3} = \text{Constant.}$$

Keplers laws were originally stated for the solar system which exemplifies them. They follow from the application of (a) Newtons Laws of Motion
(b) Newtons Law of Gravition.

We can prove keplers 3rd law for circular orbits.

$$F = \frac{GMm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\therefore T^2 = \frac{4\pi^2 r^3}{GM}$$

This is independent of the mass of satellite.

Dynamics.

Work / Power / Energy.

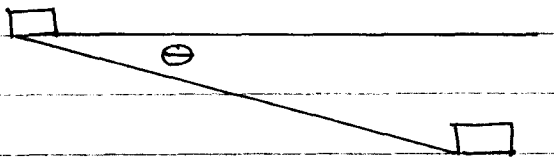
Work is said to be done when the point of application moves and is measured in the direction of the movement of the point of application.

Power is the rate of doing work.

Energy is the ability of a body to do work.

Work done = Fx but F is a vector although work Done is a scalar.

1) Work done = $F \times \cos\theta$
where θ is the angle between the 2 bodies.



(Dot product).

2. Work done = $\int_a^b F \times dx$.

3 Power = $\frac{F \times D}{T}$ = F.velocity.

4. Energy may be rotational kinetic i.e Angular Momentum, Potential Energy

Energy:

Rotational. $= \frac{1}{2} I_o \omega^2$

I_o = moment of inertia $= \Sigma \delta m x^2$
ω = angular rotation.

kinetic energy $= \frac{1}{2} m v^2$

M = mass body
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Potential energy $= mgh$.

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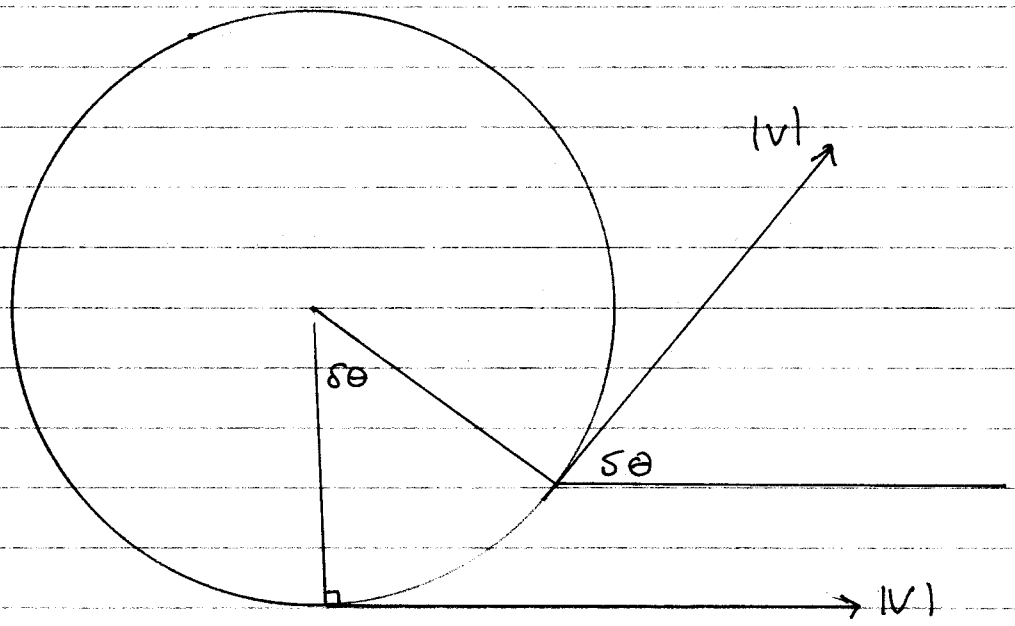
Circular Motion.



At $t=0$ $\theta=0$

velocity V horizontally

0 vertically.

Through an angle $\delta\theta$

velocity $v\cos\delta\theta$ horizontally

$v\sin\delta\theta$ vertically.

as $\delta t \to 0$ $\delta\theta \to 0$ $\sin\delta\theta \to 0$ $\cos\delta\theta \to 1$

Vertically $\frac{v\sin\delta\theta}{\delta t}$ Horizontally $\frac{v - v\cos\delta\theta}{\delta t}$

$$a = \frac{v\,d\theta}{\delta t} = v\omega.$$

Acceleration $a = rw. = r\dot{\theta}$

$w = \frac{v}{r}$

Force $= ma = mrw = \frac{mv^2}{r} = mrw^2$

Typical problems are centripedal acceleration questions.

To solve resolve vertically + horizontally

Physics Problems – Circular Motion

(Circular problems with straight forward solutions)

Do not substitute for π.

1, Find the speed and acceleration of a point moving in a circle of radius 50 cm with angular velocity (i) 4 rad/sec (ii) 300 r.p.m.

2, A point is moving round a circle at 20 m/s Find the angular velocity in rad/s and its acceleration if the radius is (i) 10 m (ii) 2m.

3, A point travels uniformly round a circle in 3 sec. (i) If the radius is 2 m, find the acceleration (ii) If the acceleration is $3\ m/s^2$, find the radius of the circle.

4, An artificial satellite goes round the earth in a circular orbit of radius r (km) in t (hr). Prove that its acceleration towards the centre is $\frac{\pi^2}{3240} \times \frac{r}{t^2}$ (m/s^2)

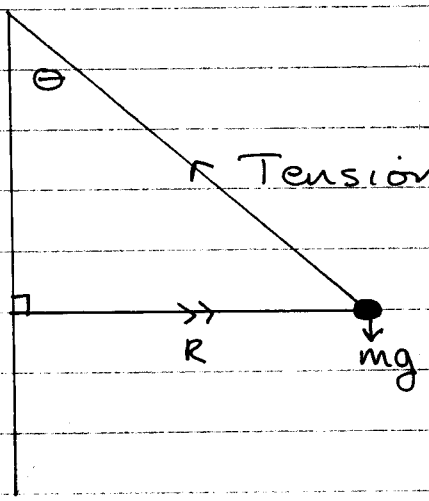
5 An aircraft is turning in a circular arc at 90 m/s. If acceleration towards the centre must not exceed 2g find the least radius in metres of the circular arc.

6, A stone of mass 2 kg attached to the end of a string is moving in a horizontal circle of radius 50 cm and completes fifty revolutions in a minute. What is the tension in the string? Show any formulae you use.

7, Flying boats at a fairground are small cars attached by chains 4 m long, whose weight may be neglected, to the ends of horizontal arms 3 m long radiating from the central pillar about which the apparatus turns. When the chains are at 30° to the vertical what is the speed of the cars?

Circular Motion

The most common problem we involving Circular motion that is set involves a weight spinning round in a horizontal circle.



The resultant of the Tension and the weight of the body is balanced by an outward force. The resultant outward force is balanced by an inward one.

Resolving Horizontally $T\sin\theta = R = \frac{mv^2}{r}$ (i)

Vertically $T\cos\theta = mg$. (ii)

Where m = mass of body

v = speed of body.

r = radius of circle described by mass m.

We may eliminate T or θ for a solution to the motion:

Either: $\tan\theta = \frac{v^2}{rg}$

or $T^2 = \left(\frac{mv^2}{r}\right)^2 + (mg)^2$

(11) Consider 2 waves: $y_1 = a\sin 2\pi f_1 t$

$y_2 = a\sin 2\pi f_2 t$

By superposition the resultant displacement is $y = y_1 + y_2$

$$y = a\left[\sin 2\pi f_1 t + \sin 2\pi f_2 t\right]$$

$$y = 2a\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t \cdot \sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

This can be split up into 2 distinct parts by comparing with wave equation $y = 2a\cos kx \sin \omega t$

$\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$ An amplitude which varies slowly with time which is the BEAT Frequency

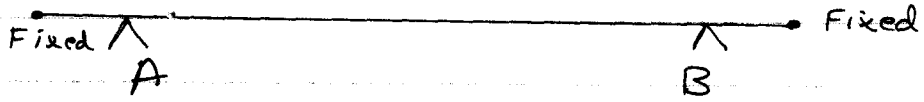
$\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$ A Displacement of freq $\left(\frac{f_1 + f_2}{2}\right)$ ranging rapidly.

Beat freq is $\left(\frac{f_1 - f_2}{2}\right) \times 2$ since the ear does not distinguish between the 2 sounds of maximum intensity i.e freq is $(f_1 - f_2)$.

Variation of Displacement is of freq $\left(\frac{f_1 + f_2}{2}\right)$

Not clear

(c)



In the experiment we cannot measure wave velocity directly but we can measure wavelength.

Using a tuning fork we strike the tuning fork and place upon a support so that the wire is constrained to vibrate at the same freq.

Then we alter length AB until measure a complete vibrating loop i.e the length of $\frac{1}{2}$ wavelength.

Using velocity = freq x wavelength

and knowing the frequency of the tuning fork used we can evaluate the velocity of the wave.

If we use a number of different tuning forks we should find the velocities calculated to be the same. ✓

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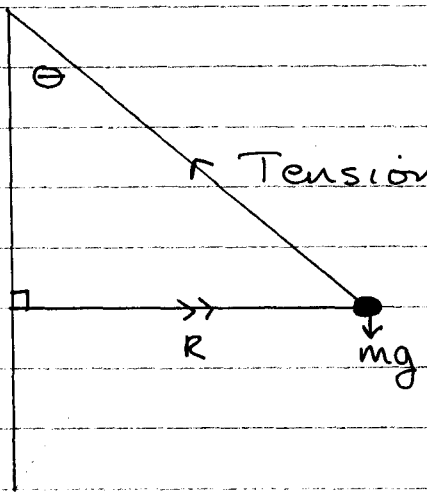

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or $T^2 = \left(\frac{mV^2}{r}\right)^2 + (mg)^2$

$$T^2 = \frac{m^2 v^4}{r^2} + m^2 g^2$$

$$T^2 = m^2 \left(\frac{v^4}{r^2} + g^2 \right)$$

$$T = m \sqrt{\frac{v^4}{r^2} + g^2}$$

Simple Harmonic Motion.

Simple harmonic motion occurs when the acceleration of a point is directed towards and is directly proportional to the distance from another fixed point.

i.e. $\ddot{x} = -\omega^2 x$

Time period $= \frac{2\pi}{\omega}$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Multiply by $2\frac{dx}{dt}$

$$2\frac{dx}{dt}\cdot\frac{d^2x}{dt^2} = -2\omega^2\frac{dx}{dt}$$

Integrating with respect to t:

$$\left(\frac{dx}{dt}\right)^2 = -\omega^2x^2 + k$$

Let $k = a^2\omega^2$ $\therefore$ $= \omega^2(a^2 - x^2)$

$$\frac{dx}{dt} = \omega\sqrt{a^2 - x^2}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \omega\, dt$$

Integrating:

$$\text{Arc}\sin\frac{x}{a} = \omega t + \epsilon$$

$$\sin(\omega t + \epsilon) = \frac{x}{a}$$

$$\underline{x = a\sin(\omega t + \epsilon)}$$

a = amplitude.
$\omega = 2\pi / T$
ϵ = phase difference.

$$x = a(\sin\omega t\cos\epsilon + \sin\epsilon\cos\omega t)$$

Let $a\cos\epsilon = A$ $\quad a\sin\epsilon = B$.

$$\underline{x = A\sin\omega t + B\cos\omega t}.$$

SHM formulae at displacement x

Velocity $v = \pm\omega\sqrt{a^2 - x^2}$
Acceleration $a = -\omega^2 x$
Kinetic Energy = $K = \frac{1}{2}m\omega^2(a^2 - x^2)$
Potential Energy = $P = \frac{1}{2}m\omega^2 x^2$

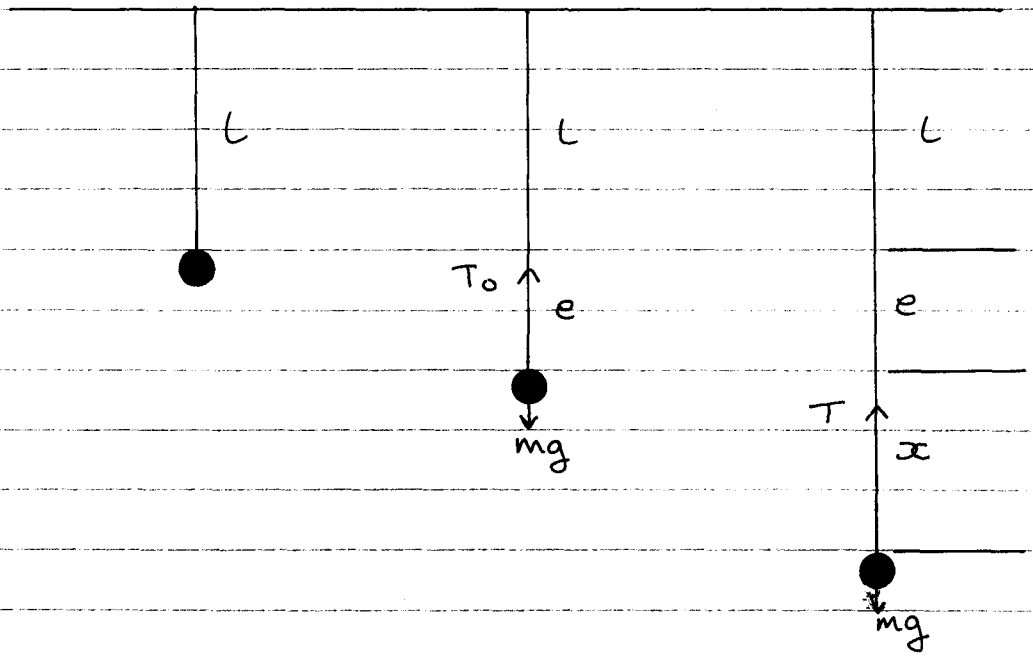
Total energy = $\frac{1}{2}ma^2\omega^2$

Simple Harmonic Motion

Simple harmonic motion occurs in a variety of circumstances and via elementry mathematical treatment it may be applied to several natural phenomena.

We may use this treatment to find out the time period T of several models.

1. Mass of a spiral spring.



Defining our variables

m = mass of body

g = acceleration due to gravity.

T_0 = Tension at equilibrium

T = Tension at a displacement x.

x = Displacement of body from equilibrium position

e = extension produced by body's own mass

L = original length of string.

λ = modulus of elasticity.

Tension = $\frac{\text{Modulus elasticity} \times \text{Displacement}}{\text{Original length}}$.

Tension $= \frac{\lambda}{L} \times$ Increase in length.

At Equilibrium

$mg = T_0 = \frac{\lambda e}{L}$

Generally: $mg - T = m\ddot{x}$

For a displacement x: $mg - \frac{\lambda(e+x)}{L} = m\ddot{x}$

$\therefore \quad -\frac{\lambda x}{L} = m\ddot{x}$

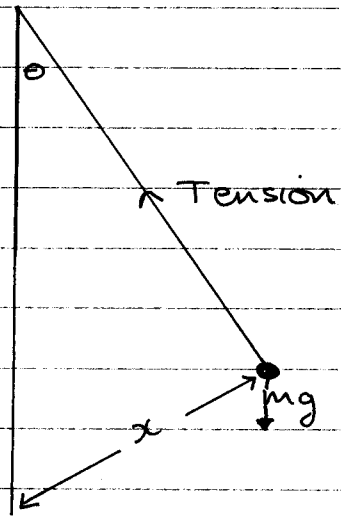
$\ddot{x} + \frac{\lambda x}{mL} = 0$ SHM.

2nd order Differential equation: $T = \frac{2\pi}{w}$

$w = \sqrt{\frac{\lambda}{mL}} \quad \therefore \quad T = 2\pi\sqrt{\frac{mL}{\lambda}}$

Simple Harmonic Motion.

Example 2 A simple Pendulum.



A mass m is placed on the end of a light inextensible string length l which is attached to a point. The body is pulled up and released.

Resolving along string

$$T = -mg\sin\theta \quad (\text{towards } \theta)$$

Perpendicular to string.

$$F = m\ddot{x} = -mg\sin\theta$$

$$\ddot{x} = -g\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots\right)$$

For small θ we may discard terms above θ^2

$$\therefore \ddot{x} + \frac{g}{l}x = 0 \qquad \text{since} \qquad \theta = \frac{x}{l}$$

SHM therefore $T = 2\pi\sqrt{\frac{l}{g}}$ $\left(\frac{2\pi}{\omega}\right)$

Leaving Cert Physics

Simple Harmonic Motion

(These will keep you bouncing with energy).

2 A small body of mass 3 kg vibrates with S.H.M. along a straight line with a frequency of 4 Hz and a maximum displacement (amplitude) of 8 cm. What is the max. speed of its motion? What is the max. kinetic energy? What is the max. potential energy?

A clock ticks everytime a simple pendulum passes through its mean position. If it ticks twice per second find the length of the pendulum to the nearest mm

3, A fishing float consists of a hollow sphere of volume $4 \times 10^{-6}\ m^3$ with a narrow stem of cross section $2 \times 10^{-5}\ m^2$. The total mass of the float + "lead weights" attached to the line is 5 g.

a) Where is the water level on the float when in fresh water?

b, If it is tugged down a little in the water and then released, show that the subsequent motion is S.H.M.

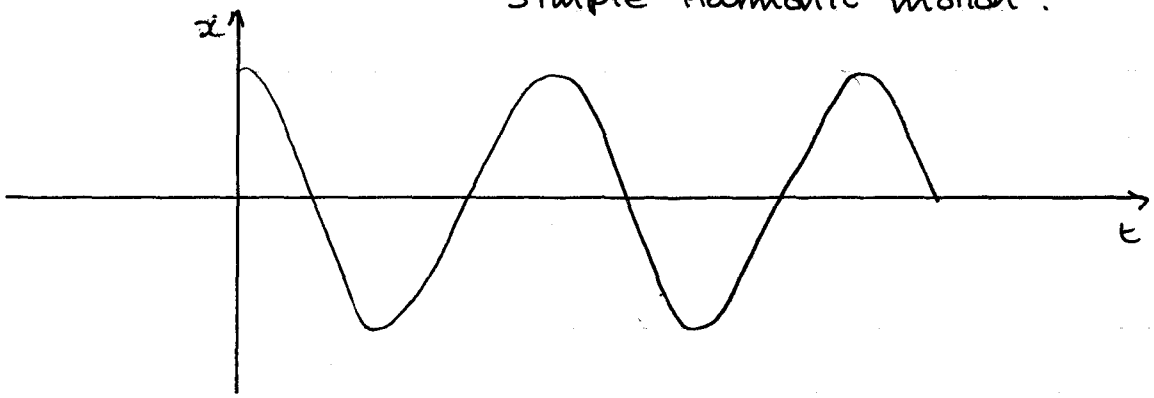
c, What is the natural frequency of oscillation of the float?

4, In an HCl molecule the force required to alter the distance between the atoms from their equilibrium separation is 540 newtons per metre. What is the fundamental frequency of vibration of the molecule assuming the vibration to be S.H.M. and the mass of the chlorine atom to be infinite compared with that of the hydrogen atom, which is 1.66×10^{-27} kg?

Physics Mechanics.

Damped Harmonic motion

1. Simple Harmonic motion.



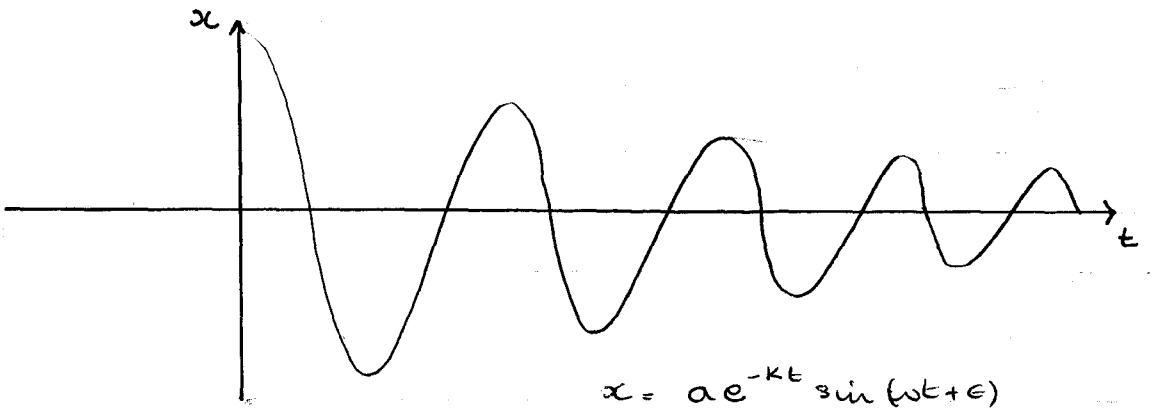
Amplitude remains exactly the same for all time.

$KE_{max} = PE_{max}$

Total energy remains constant.

$\ddot{x} + Ax = 0.$

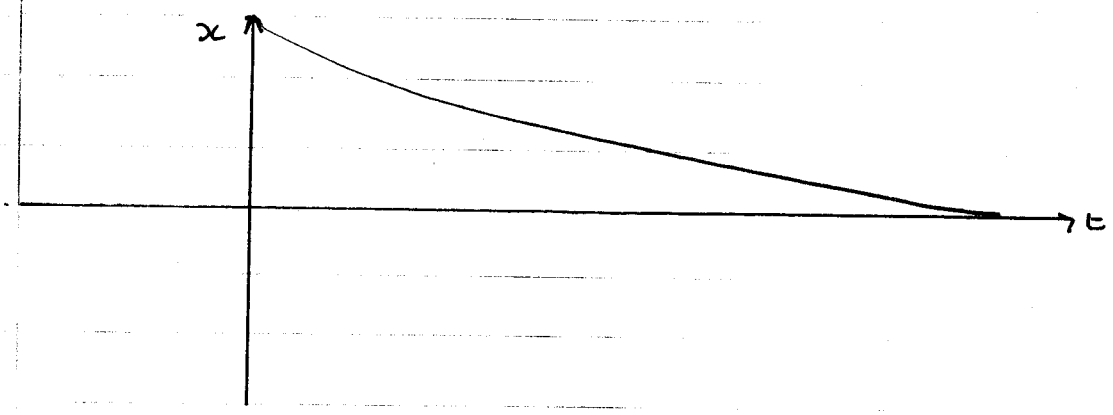
2. <u>With Slight Damping</u> (Not rec)



Amplitude diminishes with time. Energy is constantly being lost. Another force exists which is proportional to the velocity of the body.

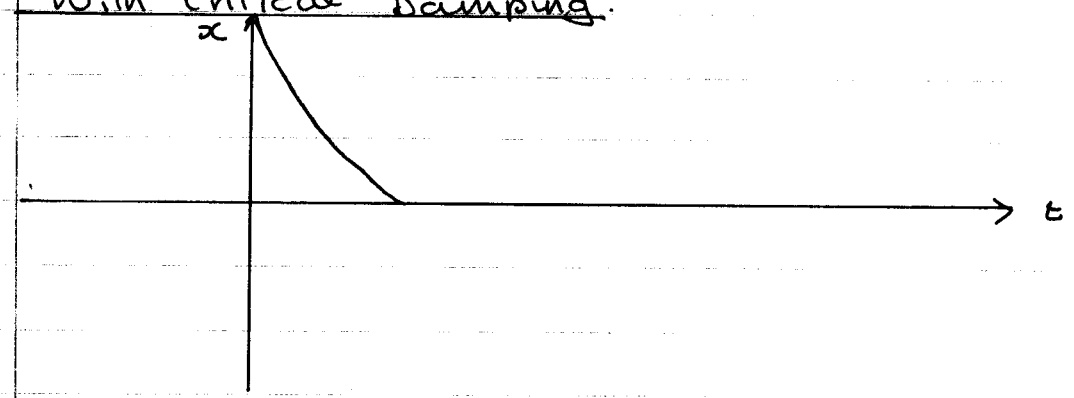
$\ddot{x} + B\dot{x} + Cx = 0$

3 With Heavy Damping.



Sometimes the damping forces are so great that no oscillatory motion occurs.

4. With Critical Damping.



The damping forces may be adjusted so that the displaced body comes to its equilibrium position in the shortest possible time. Such damping is said to be critical.

Importance and examples of damped harmonic motion.

In theory most types of harmonic ossillation would be simple harmonic but due to resistive forces of a medium or mediums most types of simple harmonic motion fall into three categories. ~~There~~ These are Underdamped, Critically Damped and Overdamped.

1. Underdamped Motion.

This usually results from light damping of a medium usually the air or water, but there are exceptions.

Simple Harmonic Motion

Examples of Simple Harmonic Motion.

1. A float on a fishing line
2. A weight attached to a helical spring.
3. A ball (which can bounce) and is dropped from a height

Examples of Critical Damping.

1. Shock Absorbers on a car.

2. Galvinometers wound on a heavy metal core
Critically damped via eddy currents.

Overdamping.

1. A meter with the terminals shorted.

Harmonic Motion as a Diff Equation.

1. Simple Harmonic Motion
$\ddot{x} + Ax = 0$
soln $x = B\sin(\omega t + \epsilon)$

2. Damped Harmonic Motion
$\ddot{x} + D\dot{x} + Ex = 0$
soln $x = Fe^{-kt}\sin(\omega t + \epsilon)$

3. Forced Harmonic Motion
$\ddot{x} + H\dot{x} + Ix = J\sin\omega t$
soln $x = L\sin(\omega t + \epsilon k)$
k is a function of the applied and natural frequency.
L is amplitude, a function of applied + nat motion of system

Forced Harmonic Motion cont.

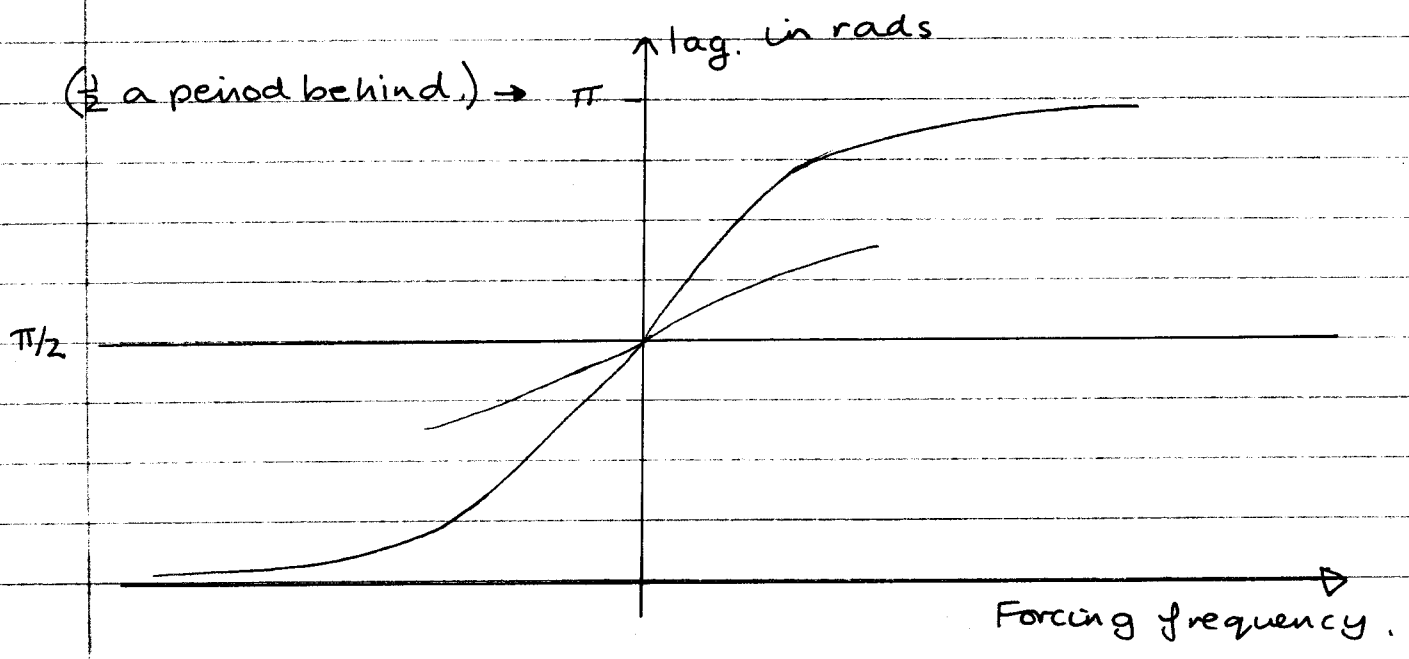
1. Energy.
The amplitude of vibration is fixed for a fixed driving frequency.

2. Amplitude.
This depends on the following:
(i) Relative values natural and driver frequency.
(ii) The damping of the system.

3. Damping shows 2 effects.
(i) Maximum amplitude occurs when the driver frequency is a little less than either damped or undamped natural frequencies.

(ii) The response is reduced of the forced system over a number of frequencies.

4. Phase.
The system adopts the frequency of the driver source but maintains a phase lag.



Advanced Level Test

Circular Motion S.H.M.

1, By considering the momentum change after it has described a small element of the circumference, show that the force required to maintain a particle of mass m travelling with uniform speed v in a horizontal circle of radius r is mv^2/r, directed towards the centre of the circle.

2, The equation of the motion for a particle moving in a straight line with simple harmonic motion can be written as $\frac{d^2x}{dt^2} = -\omega^2 x$, where x is its displacement from a fixed point in the line at time t and ω is a constant. Show that the corresponding displacement - time relation is

$$x = a \sin(\omega t + \epsilon)$$

State the meaning of the symbols a and ϵ, and obtain an expression for the velocity v at displacement x.

3, Sketch graphs to illustrate the variation of displacement and energy against time for a damped oscillating system. What happens when there is an increase in the damping factor? Illustrate the effect. The system is now made to oscillate by a variable frequency source. Again in your illustration explain the effect of variations in damping when the applied frequency is increased from zero.

Advanced

Circular Motion

1) By considering the [illegible] style it has described a small [illegible] show that the force required to [illegible] particle of mass m travelling with uniform [illegible] is a horizontal circle of radius r is directed towards the centre of the circle.

2) The equation of the motion [illegible] moving in a straight line with simple harmonic motion can be written as $\frac{d^2x}{dt^2} = -$ [illegible] where x is its displacement from a [illegible] time t and it is [illegible] that the corresponding displacement [illegible] is

$x = a \sin($ [illegible]

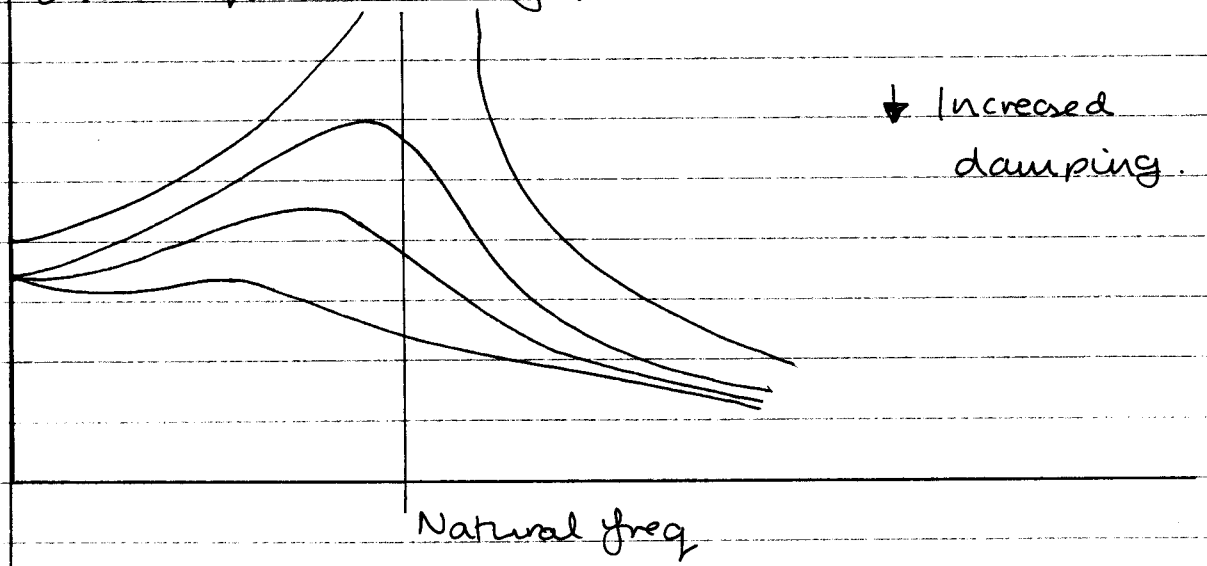
State the meaning of [illegible] and a, and obtain an expression [illegible] for the displacement x.

3) Sketch graphs to illustrate the variation of displacement and energy [illegible] for a damped oscillating system. What happens [illegible] in the damping [illegible] system is [illegible] frequency [illegible] the effect of resonance [illegible] frequency is [illegible]

Harmonic Motion continued

5. Resonance.

Resonance occurs when the frequency of the driver equals that of the driven.



For Absorbsion / Emission spectra
see Chemistry Notes.

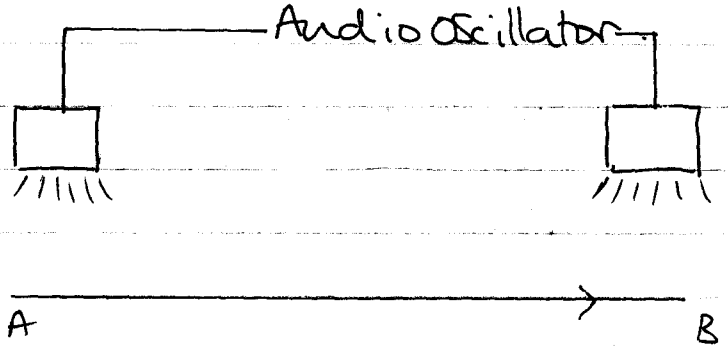
Pagel. M. Bennett.

4 1974.

Sound may be demonstrated to be a wave motion using a simple laboratory experiment. The experiment demonstrates the soundwaves ability to interfere and superpose a property by definition of wave motion. ✓

In the ~~eg~~ experiment an audio oscillator feeds 2 identical speakers placed about 2 metres apart. If an observer walks from one end to the other end of the apparatus he will percieve alternate loud and quiet sound of the same frequency. This is constructive and destructive superposition resulting from the inteference of the WAVES. ✓

i.e



(Observer walks from A to B)

(11) Consider 2 waves: $y_1 = a\sin 2\pi f_1 t$

$y_2 = a\sin 2\pi f_2 t$

By superposition the resultant displacement is $y = y_1 + y_2$

$$y = a\left[\sin 2\pi f_1 t + \sin 2\pi f_2 t\right]$$

$$y = 2a\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t \cdot \sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

This can be split up into 2 distinct parts by comparing with wave equation $y = 2a\cos kx \sin \omega t$

$\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$ An amplitude which varies slowly with time (1/2x) is the BEAT Frequency

$\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$ A Displacement of freq $\left(\frac{f_1+f_2}{2}\right)$ ranging rapidly.

Beat freq is $\left(\frac{f_1 - f_2}{2}\right)\times 2$ since the ear does not distinguish between the 2 sounds of maximum intensity i.e freq is $(f_1 - f_2)$.

Variation of Displacement is of freq $\left(\frac{f_1 + f_2}{2}\right)$

Not clear

Vibration + Waves.

2 types of waves. (Progressive)

1. Longditudional Waves.
Here the vibration of the mechanical particles is parallel of translation of energy.
Example Sound Waves.

2. Transverse Waves.
The vibration of particles is perpendicular to the direction of energy translation. These waves may be plane polarised unlike Longditudional waves.
Example Water waves.

Definitions.

Amplitude of a wave is its maximum displacement from its mean position.

Wavelength is the distance between two particles of a wave having the same phase.

Frequency is the number of complete oscillations per second.

Velocity is the rate of change of distance moved with time

Power of a Wave Motion.

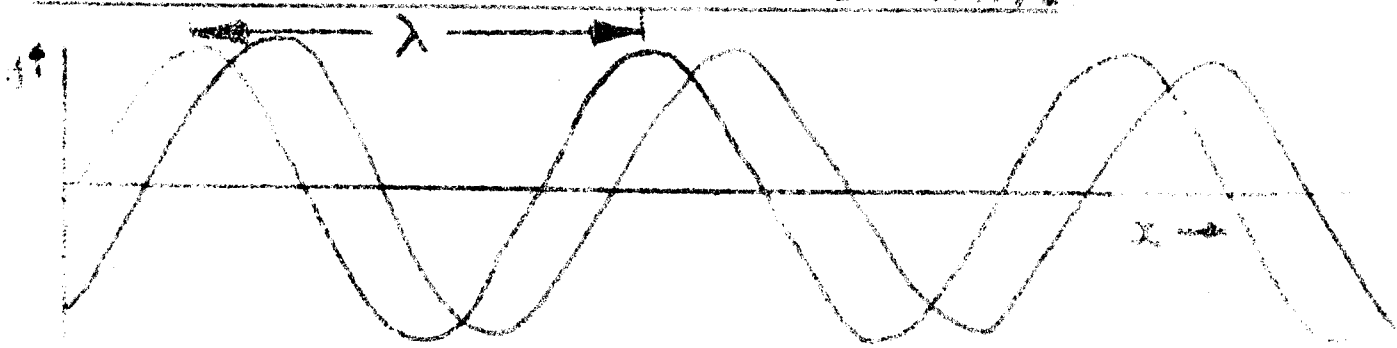
Wave motion transfers momentum and energy.

Energy $W = \frac{1}{2} \lambda \mu a^2 \omega^2$

λ = length
μ = mass per unit length

Power = $\frac{W}{T} = \frac{1}{2} c \mu a^2 \omega^2$

EQUATION OF A PROGRESSIVE WAVE



The diagram represents two instantaneous photographs of a [illegible]. The blue trace is the appearance of the [illegible] at time $t = 0$.

[illegible] before the wave motion takes [illegible] the red trace, which it does by a time [illegible] the wave outline has progressed a distance [illegible] given by the relation

$x = vt$ where v is the velocity [illegible]

[illegible] the origin O at a point P back on the [illegible] a distance [illegible]

[illegible] at $t = 0$ by a fraction [illegible] of the time period,

$\frac{x}{vT}$ of a cycle,

[illegible] the wave to travel at time

$$y = A \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)$$

Superposition of Waves

The principle states:
The net disturbance at a given place and time caused by a number of waves is the vector sum of the amplitudes of the individual waves

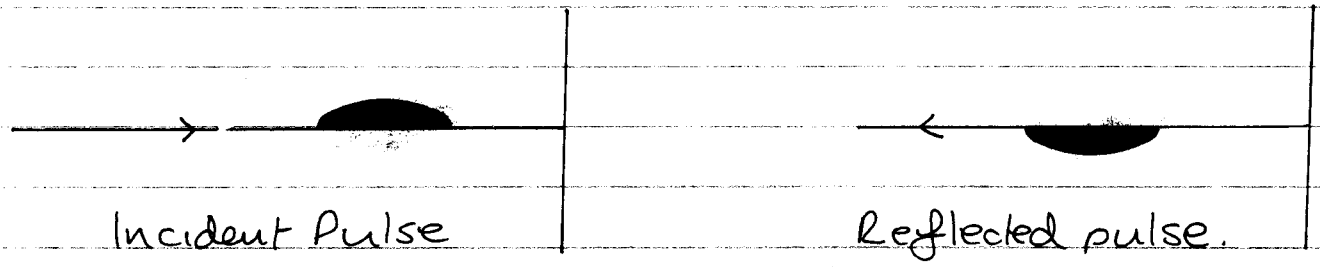
Superposition occurs in
1. Stationary Waves — Beats.
2. Interference — Diffraction.

1. Stationary Waves.

Stationery or standing waves result from the superposition of 2 wavetrains of equal amplitude and frequency but of opposite velocity.

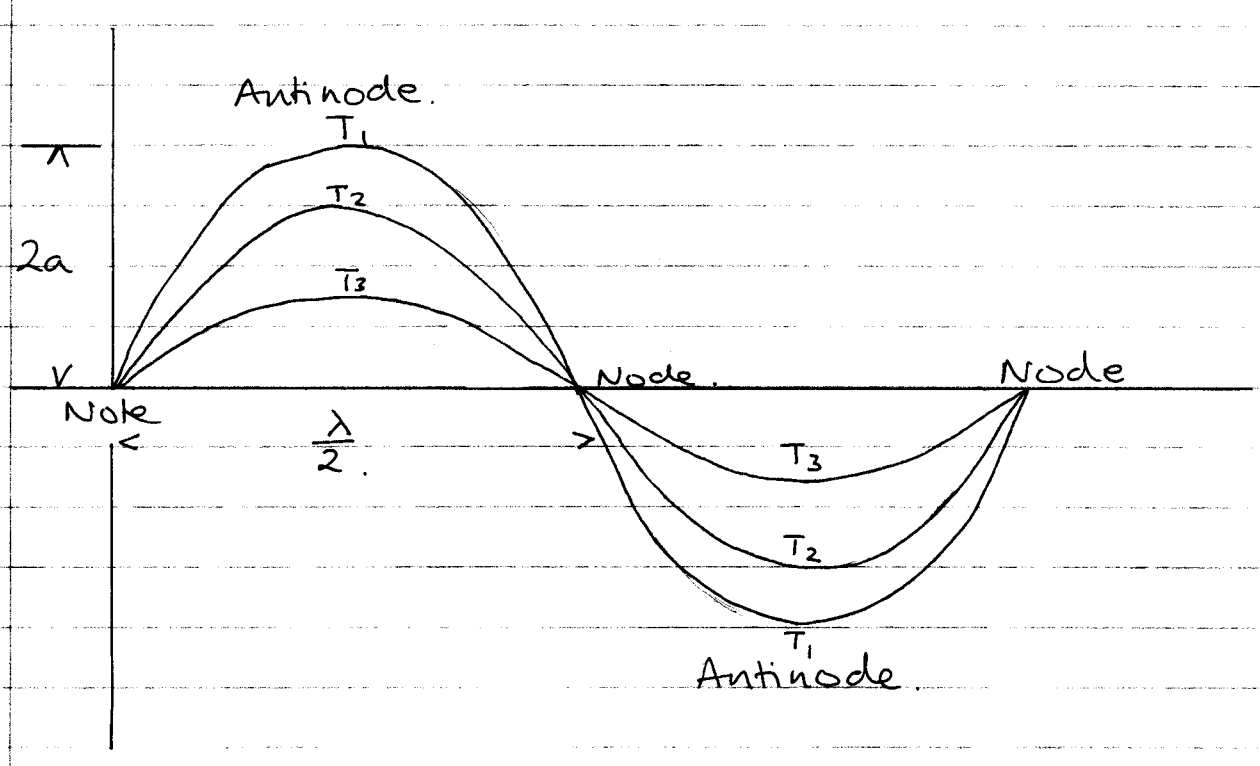
Reflection of a Transverse wave Pulse

At a fixed boundry such a wave experiences a Phase change of π radians. We may explain this by saying that an upward pulse incident on a fixed object pulls upwards. The reaction at the wall will be downwards + hence there is a phase change of π radians.



We may use the fact that there is a π rad phase change at a fixed boundry to produce standing waves.

Here is a typical standing waves.
Points of zero displacement are Nodes.
Points maximum displacement Antinodes.



We can describe the properties of standing waves best by comparing their properties with that of a Progressive wave which may be either Longditudional or Transverse.

A Comparison/Summary of Properties of Stationary + Progressive Waves.

Question	Stationary	Progressive.
Amplitude	Varies from 0-2a depending upon position	Always constant a anywhere on wave.
Frequency.	All particles vibrate with same frequency in SHM.	All particles vibrate in SHM with freq of wave.
Phase	All particles in $\frac{1}{2}$ have same phase.	only other wavelengths in phase.
Waveform.	Does not advance Appears stationary.	Advances with velocity of wave.
Energy	Although the wave has energy there is NO translation of energy.	Energy translation occurs in the direction of the wave.

Mathematical Treatment of Standing Waves

We have 2 waves y_1 y_2. The equation of the standing wave by superposition is $y = y_1 + y_2$

$y_1 = a\sin(\omega t - kx)$ to the right

$y_2 = a\sin(\omega t + kx)$ to the left

$$y_s = y_1 + y_2 = a\left[\begin{matrix}\sin\omega t\cos kx - \sin kx\cos\omega t \\ +\sin\omega t\cos kx + \sin kx\cos\omega t\end{matrix}\right]$$

$$y = 2a\sin\omega t\cos kx$$

$y =$ (time dependant)(location dependant)

Deductions from the above equations.

1. Time period $= \omega = \frac{2\pi}{T} = 2\pi f$.

2. At nodes $y = 0 \Rightarrow \cos kx = 0$
   $\cos = 0$ every π rads i.e every $\lambda/2$ metres.

3. Oscillation amplitude varies with location
   i.e the $\cos kx$ term $(k = 2\pi/\lambda)$

4. At antinodes $y = 2a$ i.e $\cos kx = 1$ i.e every π rads or $\lambda/2$ metres.

5. No net energy transfer.

Waves (Mechanical).

We can derive dimensionally an expression the velocity of a wave

		Dimensions:
Velocity	V	$ML\,T^{-1}$
Tension	T	MLT^{-2}
Mass/length	M	ML^{-1}

$V = K(\text{onstant})\ T^x M^y$

Equating coefficients: $0 = x + y$ $\quad 1 = x - y$

$\Rightarrow x = \frac{1}{2}$ $\quad y = -\frac{1}{2}$

$$\text{Velocity} = k\sqrt{\frac{T}{M}}$$

If the wave is constrained between 2 fixed supports then a standing wave may be generated if a whole number of HALF wavelengths fit between the supports. $\left[L = \frac{n\lambda}{2}\right]$

By suitable experiment we find that k the constant of proportionality to be 1

$$v = \sqrt{\frac{T}{M}}$$

Knowing however $v = f\lambda$

$$\Rightarrow f = \frac{v}{\lambda}$$

Thus $f = \frac{1}{\lambda}\sqrt{\frac{T}{M}}$

FOR A STANDING WAVE

L = length between fixed supports $= \frac{n\lambda}{2}$

$\Rightarrow \frac{1}{\lambda} = \frac{n}{2L}$ $\qquad f = \frac{n}{2L}\sqrt{\frac{T}{M}}$

For a standing wave $f = \frac{n}{2L}\sqrt{\frac{T}{M}}$

Redefining above variables.

n = number of loops (= harmonic)

i.e $n=2$ = 2nd harmonic (1st overtone).

L = length between supports.

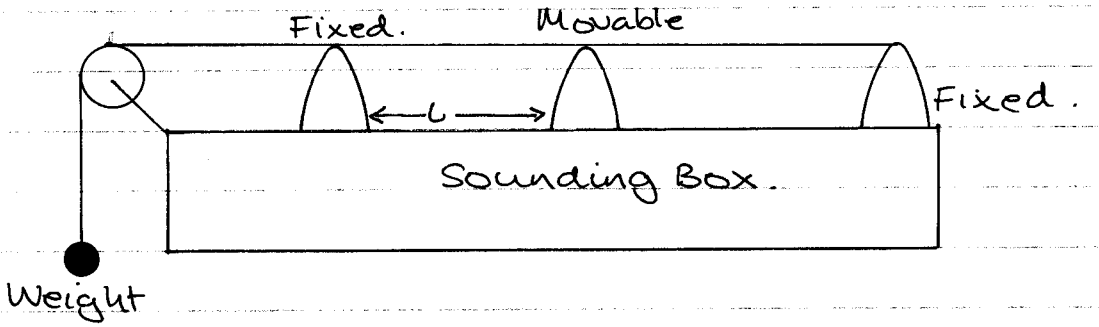
T = Tension in string.

M = Mass per unit length of string.

Waves continued.

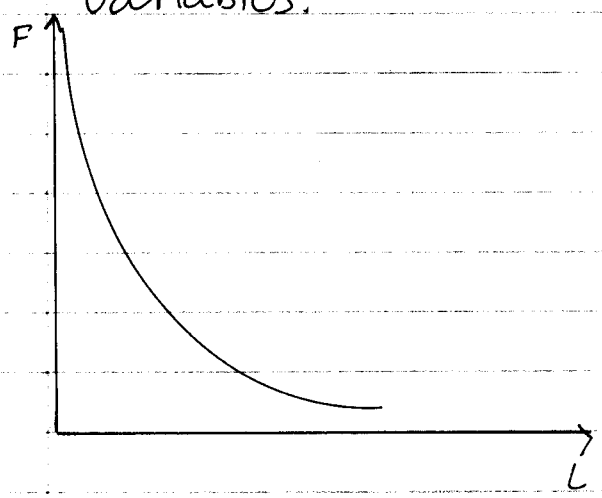
The Sonometer:

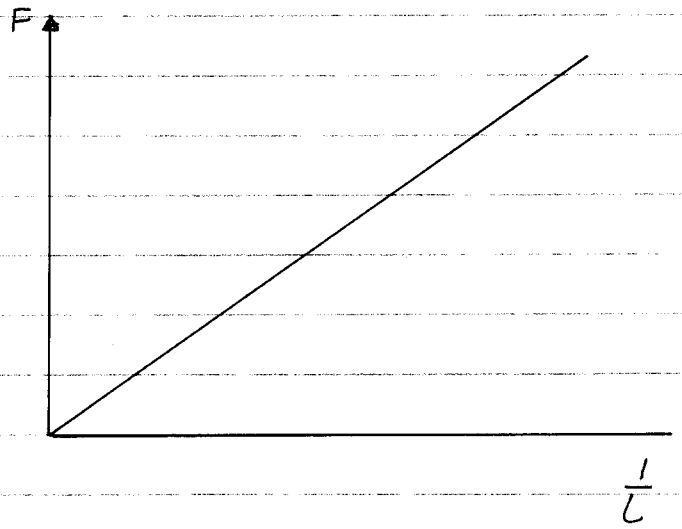
We can use the sonometer to check & evaluate $f = \frac{n}{2L}\sqrt{\frac{T}{M}}$ and also work out other relations.

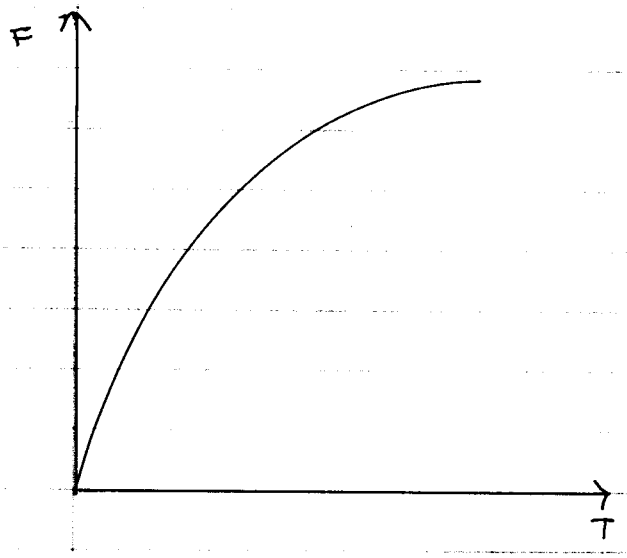


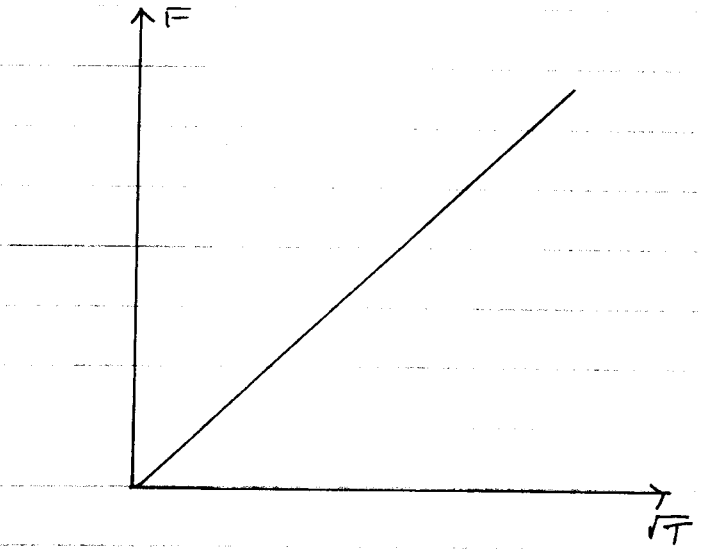
L = length of vibration
T = Tension of wire.
M = Mass per unit length of string.

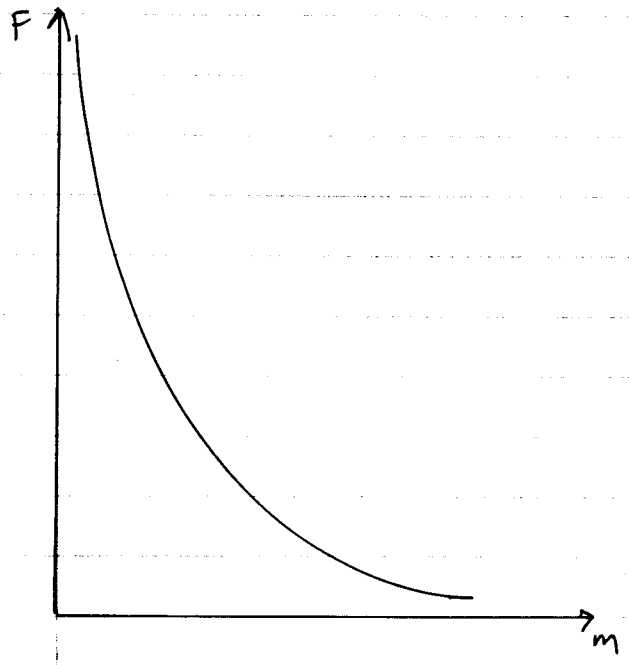
We may vary different parameters and hence find out what the relationship between the above variables.

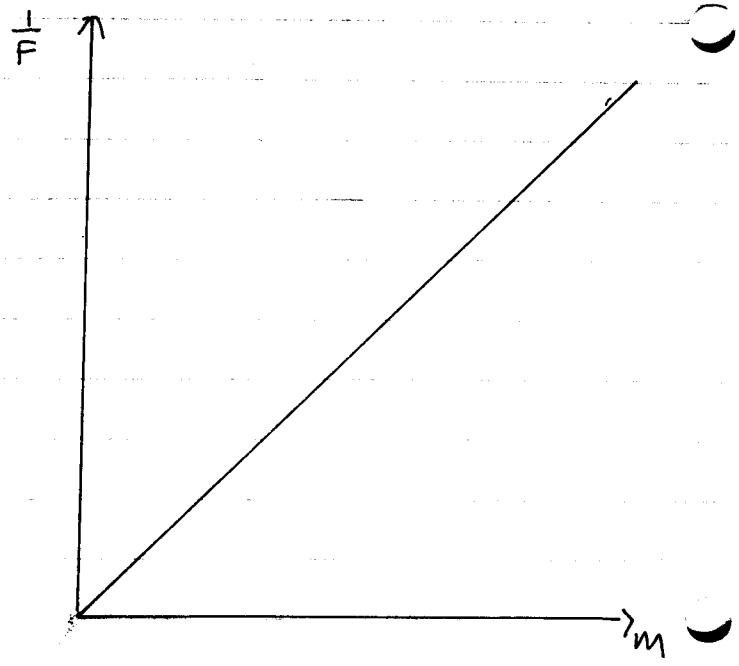












$F = \frac{1}{2L}\sqrt{\frac{T}{m}}$. For n loops we have

$F = \frac{n}{2L}\sqrt{\frac{I}{m}}$

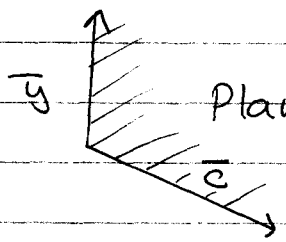
Polarisation of Waves.

This is a transverse of transverse waves. whereby the wave motion is restricted to 2 dimensions only.

2 vectors must be quoted.

(i) wave velocity $\bar{c}$

(ii) particle displacement $\bar{y}$



examples include electromagnetic waves.

We choose the plane containing the velocity vector $\bar{c}$ and the electric vector $\bar{E}$ to be the plane of polarisation.

Examples of Wave Polarisation.

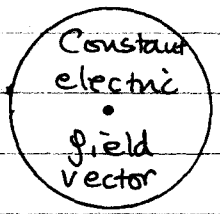
1. Microwave transmitter

Emmiter, receiver have metal rod for such. For maximum signal recieved 2 ~~aerie~~ aerials must be parallel. (Minimum if at 90° to each other)

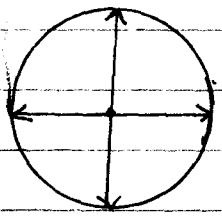
If a grid of parallel bars is placed in front of the transmitter it will absorb the signal leaving the reciever with no signal if its bars are parallel to transmitter.

Light from a filament or gas discharge tube is said to be unpolarised i.e. that Photons of all possible polarisations are emmited.

This occurs because the excited atoms which give out the light are in random orientation (unlike the electrons in a radio aerial). This is represented by :-

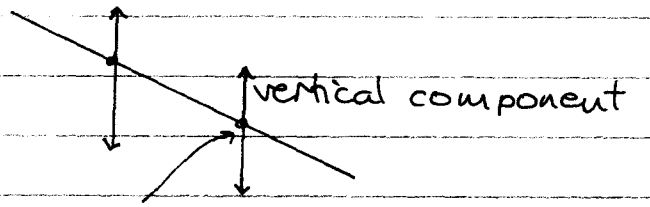


End view of a ray.



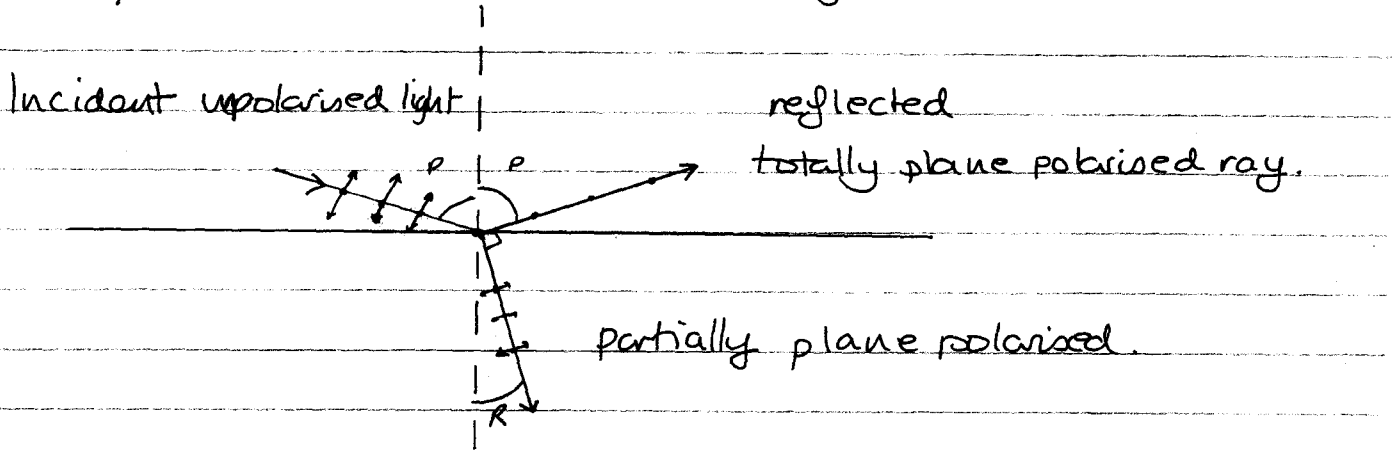
We may resolve into 4 components.

or (II)



Brewsters Angle.

Some polarisation occurs for any reflection or refraction Complete linear polarisation of the reflected wave only occurs at Brewsters Angle.



Brewsters Angle continued.

$\frac{\sin P}{\sin R} = n$ i.e refractive index.

Know $R + P = 90°$, $R = 90 - P$

$$n = \frac{\sin P}{\sin(90-P)} = \frac{\sin P}{\cos P} = \tan P$$

Defining Variables

n = Refractive index of medium

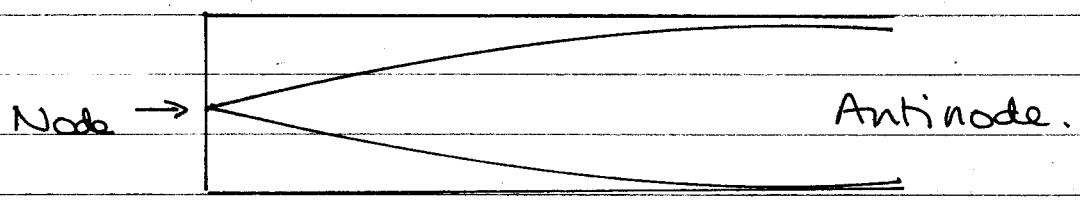
P = Angle at which polarisation occurs
$\doteq$ Brewsters Angle.

$\underline{n = \tan P \text{ is Brewsters Law.}}$

~~This~~ The above result is important since it enables the evaluation of the refractive index of opaque samples.

It is particularly useful since no refracted ray need be found.

Sound Waves in Pipes.



Fundamental frequency in a pipe

Pipe length l wavelength $\frac{\lambda}{4}$

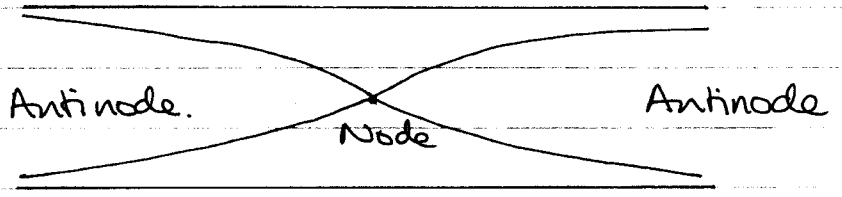
Must be an Antinode at open part of pipe.

Next harmonic is the 3rd. To calculate this see how many $\frac{1}{4}$ wavelengths are produced. Here the frequency is $\frac{3}{4}\lambda$ thus 3rd Harmonic.

Next harmonic is 5th Harmonic.

A closed pipe thus gives odd harmonics only.

Open Pipes.



Fundamental wavelength $\frac{\lambda}{2}$ in length L.

Next harmonic is the second in the first overtone. ~~Fa~~ Next wavelength is $2\lambda/2$ is λ This is the second harmonic.

Next harmonics are third and fourth.

An open pipes recieves all harmonics.

Beats.

2 frequencies ~~f_1~~ , f_2.

No cycles of f_1 in t is $f_1 t$
f_2 in t is $f_2 t$

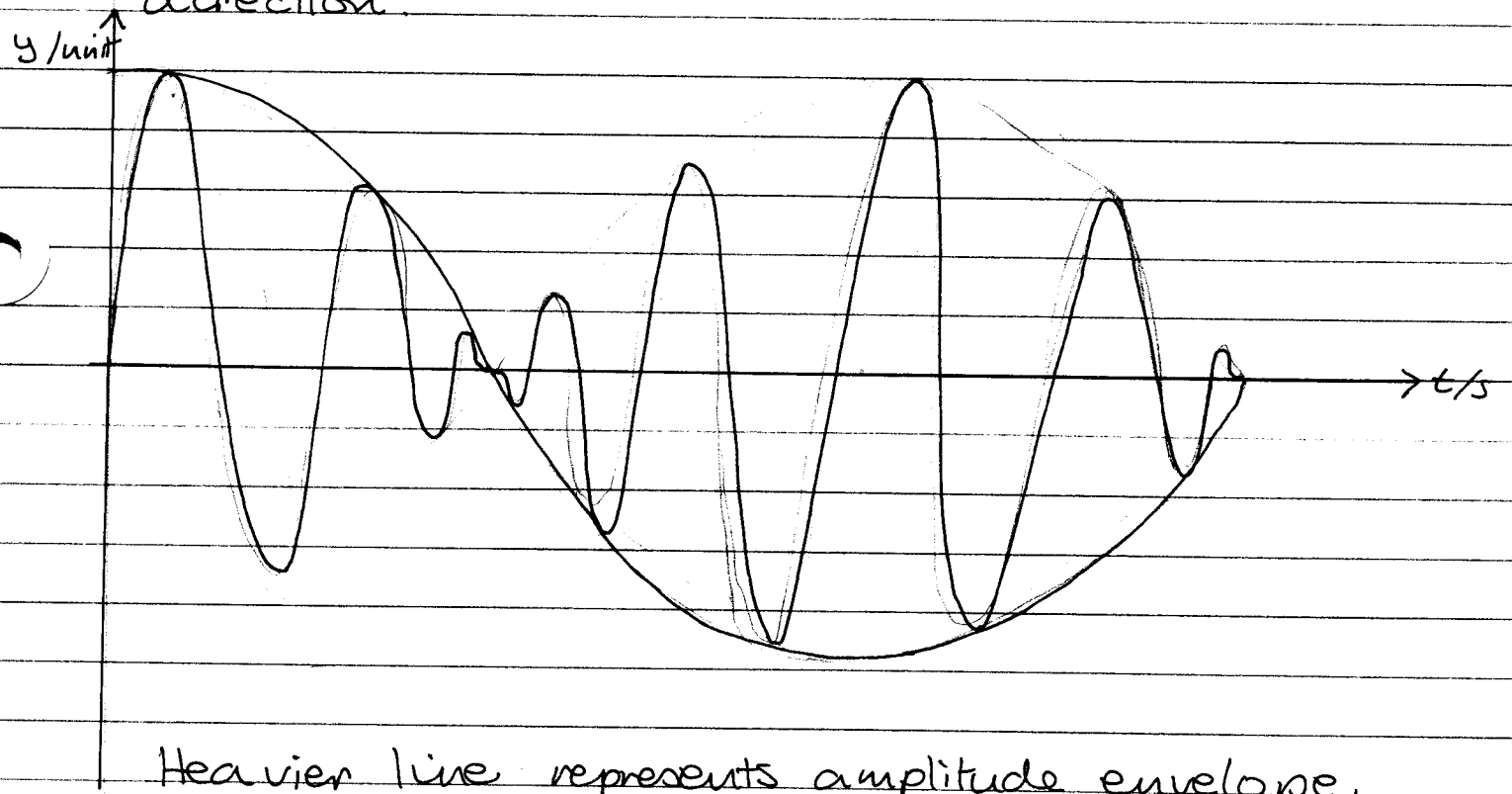
$(f_1 - f_2)\, t = 1$ for beats.

$\frac{1}{t} = f = f_1 - f_2$

∴ Beat frequency = $f_1 - f_2$

Beats.

Beats occur when 2 wavetrains of slightly different frequencies but equal amplitudes superpose whilst travelling in the same direction.



Heavier line represents amplitude envelope.
Thin line shows actual beat frequency.

If we have 2 waves.

$$y_1 = a \sin 2\pi f_1 t \qquad y_2 = a \sin 2\pi f_2 t$$

By superposition Resultant displacement

$$y = y_1 + y_2$$

$$y = a\left[\sin 2\pi f_1 t + \sin 2\pi f_2 t\right]$$

$$= 2a \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) t$$

$$y = 2a \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \quad \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) t$$

$$y = \pm 2a \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) t \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t$$

$y =$ [Displacent of freq $\left(\frac{f_1 + f_2}{2}\right)$ varies rapidly with time] $\times$ [An amplitude which varies at freq $\left(\frac{f_1 - f_2}{2}\right)$ slowly with time]

<u>Calculation of Beat frequency</u>

Let there be a beat period T

Wave freq f_1 completes 1 more cycle than f_2

$$\therefore f_1 T - f_2 T = 1$$

$$f_{beat} = \frac{1}{T} = f_1 - f_2$$

Doppler.

This is the apparent change in frequency when there is relative movement between source and observer.

2 cases.

(1) Observer moving towards stationary source.

Wavelength of emission is unchanged

$$\text{Wavelength emission} = \frac{v}{f}$$

Observer moves at speed u so velocity of waves relative to observer $= (v + u_o)$

$$\text{Apparent frequency} = \frac{\text{relative velocity of waves}}{\text{wavelength}}.$$

$$f_o = \frac{v + u_o}{v/f} = \left(\frac{v + u_o}{v}\right) f$$

Observer moves away $= \left(\frac{v - u_o}{v}\right) f$

$$\text{Apparent frequency} = \left(\frac{v \pm u_o}{v}\right) f \quad \begin{matrix} \text{towards} \\ \text{away} \end{matrix}$$

(ii) Source Moving towards observer.

When source at rest f waves at velocity v emitted. Source moving at u_s then in 1 second $(v - u_s)$ represents the distance moved by f waves.

$$\text{New wavelength} = \lambda_o = \frac{(v - u_s)}{f}$$

$$F_o = \text{Apparent frequency} = \frac{\text{Velocity of Waves}}{\text{Apparent wavelength}}$$

$$= \frac{v}{(v-u_s)/f} = \left(\frac{v}{v-u_s}\right) f.$$

$$f_o = \left(\frac{v}{v-u_s}\right) f$$

$$\text{Source moving away} = \left(\frac{v}{v+u_s}\right) f$$

$$\text{Frequency } f_o = \left(\frac{v}{v \mp u_s}\right) f \quad \begin{matrix} S \text{ towards} \\ \text{away} \end{matrix} \; O$$

Summary.

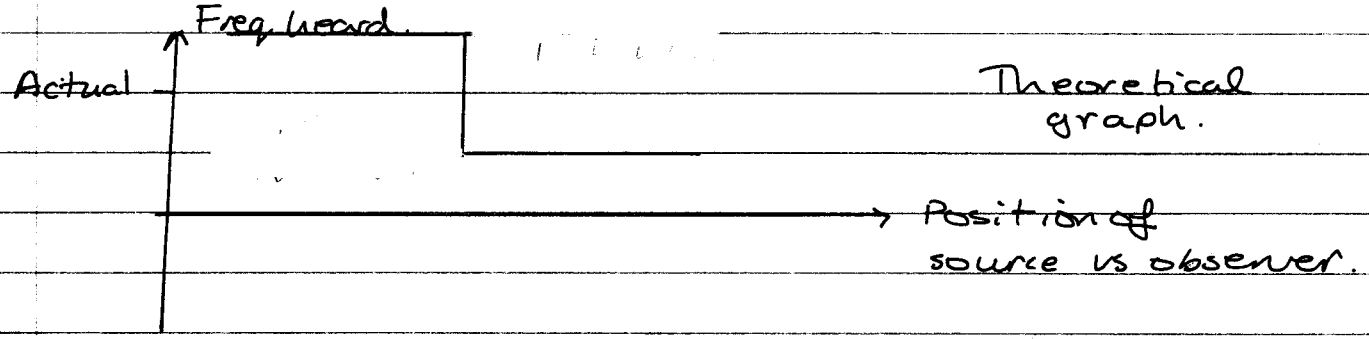
$$f_o = \left(\frac{v \pm u_o}{v \mp u_s}\right) f \quad \begin{matrix} O \text{ towards} \\ \text{away} \end{matrix} \; S \qquad \begin{matrix} S \text{ towards} \\ \text{away} \end{matrix} \; O$$

Velocity of Light : Duncan/DVP
Rot Mirror.

Physics Mechanics.

Doppler continued

1. Moving source past a stationary observer



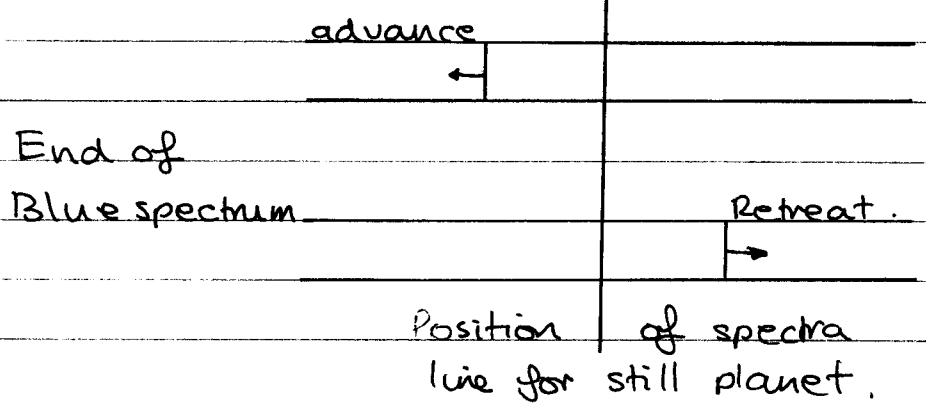
As source approaches a higher than normal frequency occurs which changes very quickly as the source passes the observer to a lower than normal frequency.

2. The Red Shift.

A retreating light source causes any particular spectral line received from that source to be shifted towards the Red part of the spectrum.

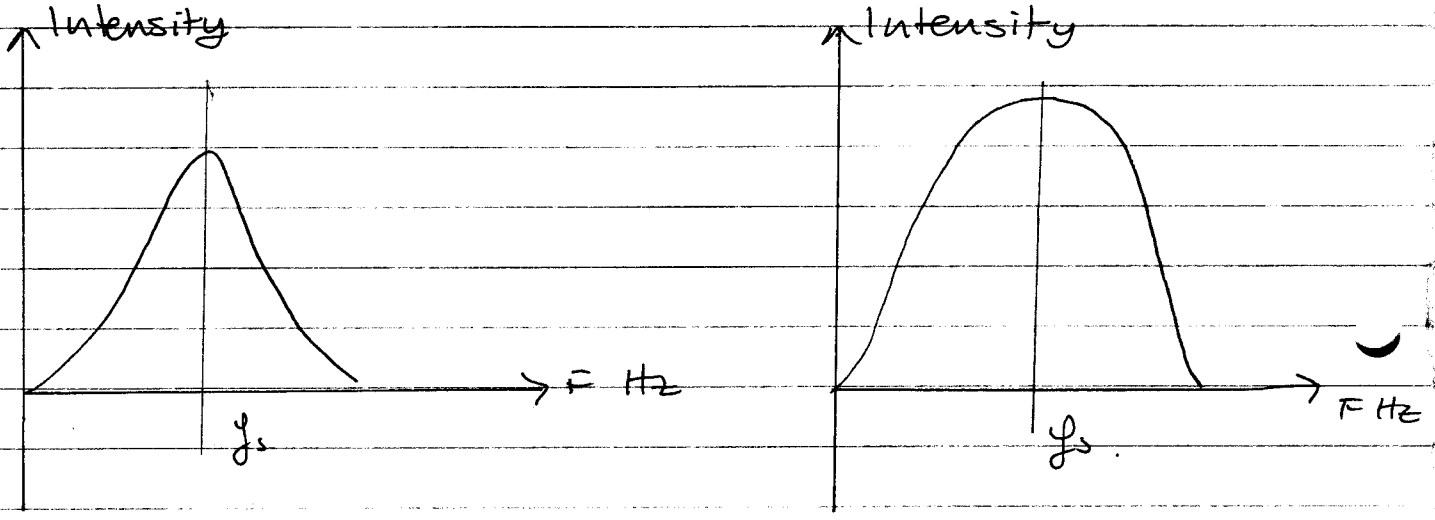
Speed of recession $v = Hr$ where r is the distance away from the earth, H is Hubble's constant.

Red shift can be represented diagramatically.



3 Doppler Broadning.

Gas molecules emitting light have vaniable velocities resolved along the line of observation. This results in a ~~bro~~ broadning of spectral lines.



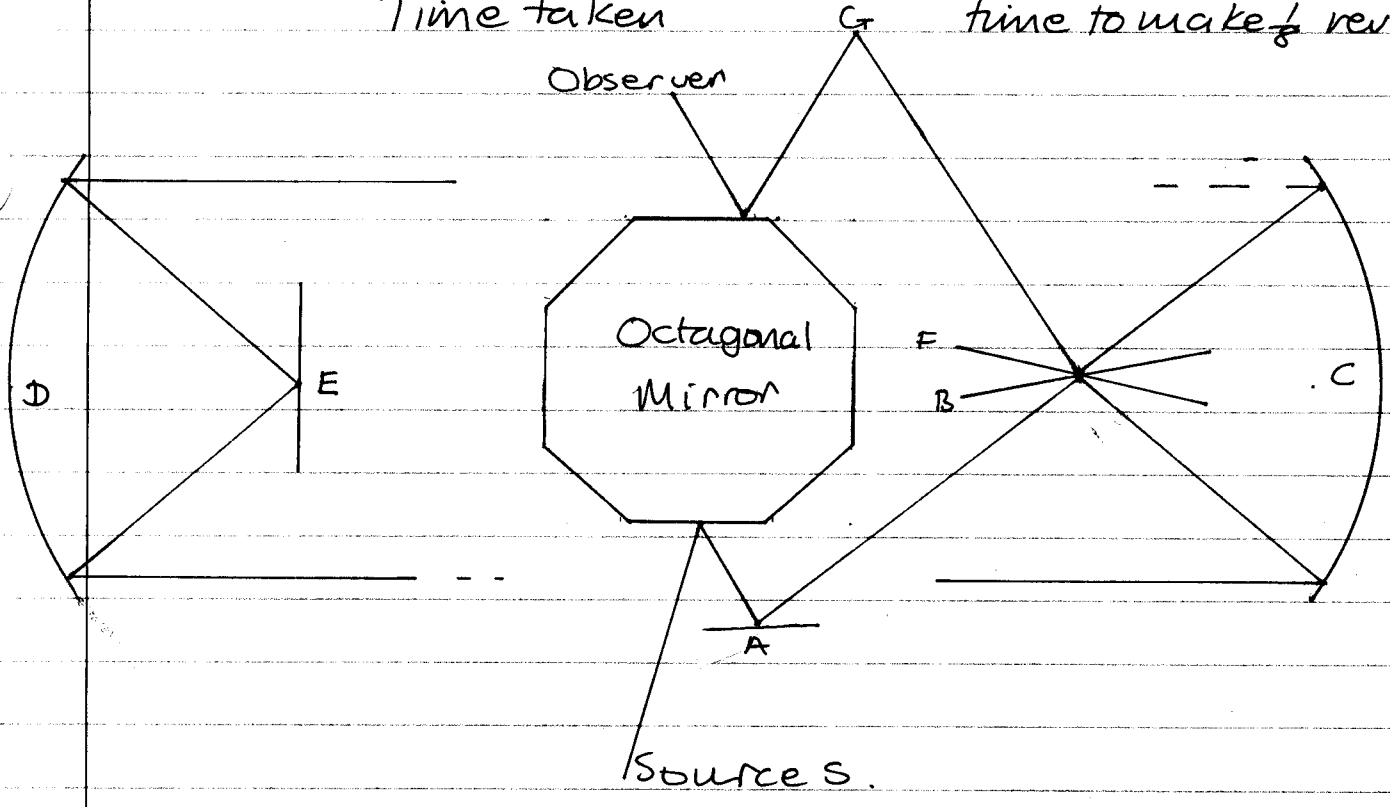
Determination of the Speed of Light

We use Michelson's Method for Determining the speed of light. The light from the source follows the path A, B, C, D, E, D, C, F, G octagonal mirror eyepiece.

During this time the mirror is made to rotate exactly one eighth of a revolution so that an image is seen in the eyepiece. For slightly higher or lower rates of rotation no image will be seen.

The rotational frequency of the mirror can be measured Stroboscopically.

$$\text{Speed} = \frac{\text{Distance Travelled}}{\text{Time taken}} = \frac{2 \times \text{mountain sep}}{\text{time to make } \frac{1}{8} \text{ rev}}$$



Diffraction

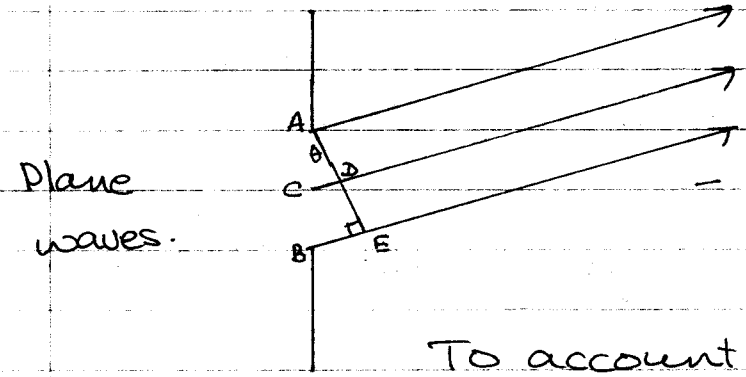
Process of the spreading out of light into a Geometric shadow region.

When light shines through a slit and diffraction occurs the degree of its effect is dependent upon the slit size

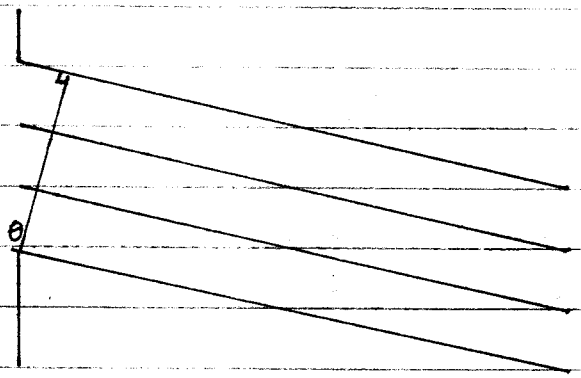
(i) A Broad slit does not show much diffraction except at light edges where path becomes slightly broader.

(ii) Medium slit
A semi circle of light produced.

(iii) Small slit
Diffraction pattern obtained. We can explain this using Huygens construction.



To account for the maxima and minima so produced we use Huygens construction. For the above all the light from AC is cancelling out by superposition light from CB. For this to be true $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ $\therefore \sin\theta = \frac{\lambda}{a}$ for first minima.



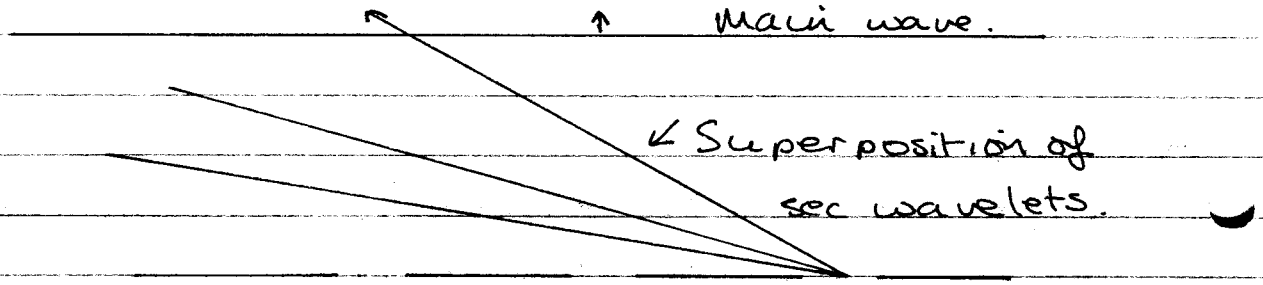
Here $\frac{a}{3}\sin\theta = \frac{\lambda}{2}$ $\sin\theta = \frac{3}{2}\frac{\lambda}{a}$.

Here a maximum is obtained since uneven number of light strips.

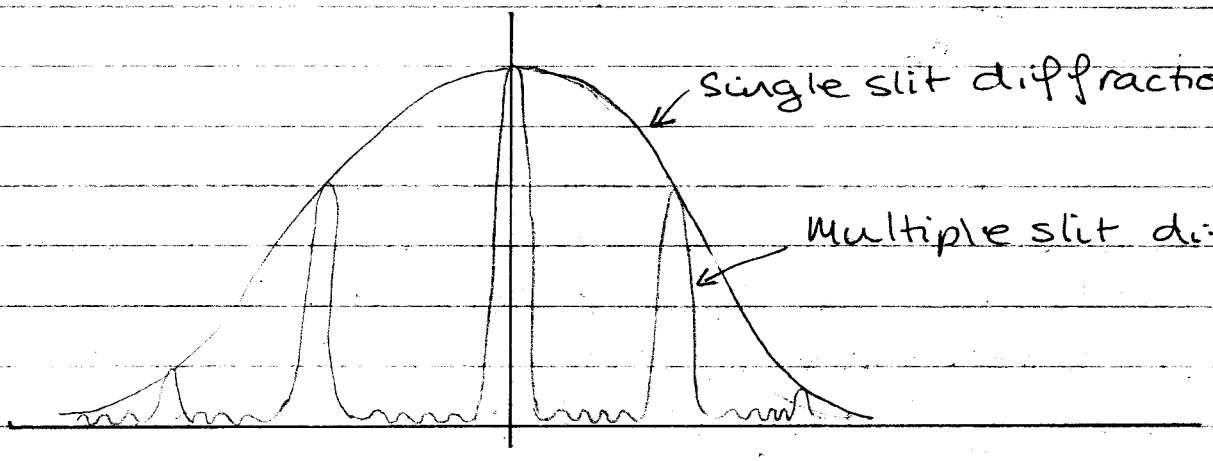
By a pairing process minima will occur at

$$\sin\theta = \frac{n\lambda}{a}.$$

Diffraction at Multiple slits.

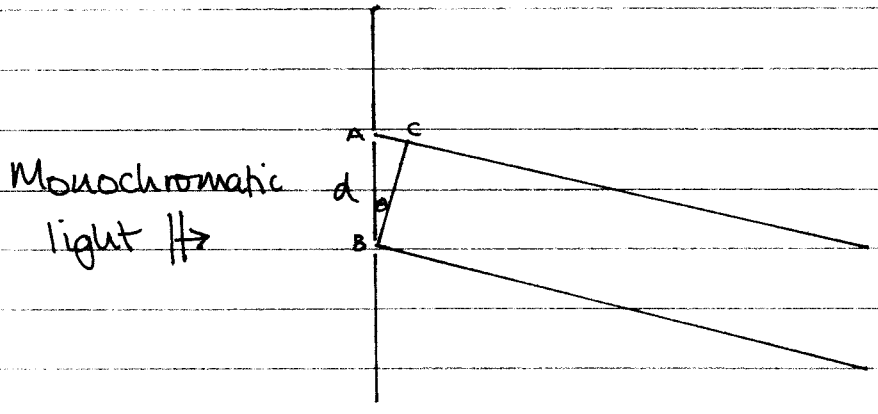


As number slits increases pattern changes TO one similar to a diffraction grating.



Diffraction gratings.

Light falls upon a diffraction grating of wavelength λ and slit seperation d



dist $d = AB$.

if $d \sin\theta = \lambda$ then constructive superposition occurs. λ may be replaced by $n\lambda$ as no phase change thus $n\lambda = d\sin\theta$.

$$\sin\theta = \frac{n\lambda}{d}$$

Eye can see:

0·4 ·5 ·6 0·7 μm.
Violet Blue Green Yellow. Red

M. Bennett

Light.

Newtons Corpuscular Theory.

Newton studied reflection and ~~reflectia~~ refraction of light in a medium and produced the Corpuscular Theory. He assumed light consisted of small corpuscles which needed a medium to travel in. He said that in collision they had no optical friction and performed perfectly elastic collisions.

For Reflection perfectly elastic boundry collision occurs and particle rebounds $i = r$

For Refraction the particle bends towards normal because the perpendicular component to surface of collision increases.

Newtons theories were proved wrong because

(1) He said light needed a medium to travel in

(2) The theory requires an increase in the speed of light in a dense medium. The reverse is found to be true.

Huygens Construction.

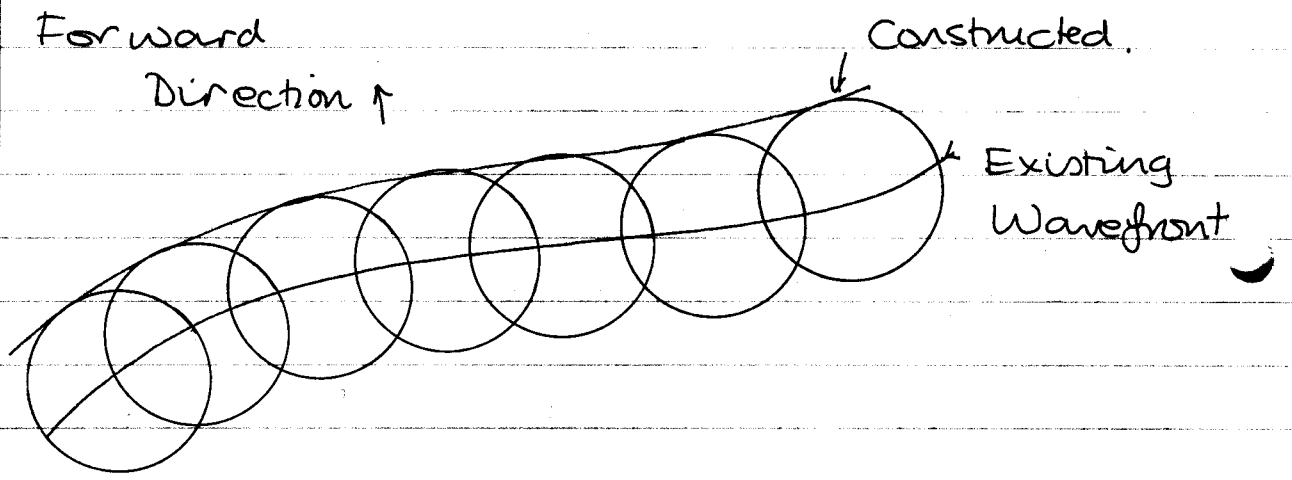
This is not a fundamental physical principle but a useful mathematical technique.

Theory.

Each point on an existing wavefront can be considered as a source of secondary wavelets.

(i) The wave amplitude at any point ahead can be obtained by the superposition of these wavelets.

(ii) From a given position of an uninterupted wavefront a later position can be determined by the envelope of the secondary wavelets.



Example of Huygens Construction

Page 101 EPS for Applications to Reflection + Refraction.

Diffraction.

Diffraction is the result of superposing light waves from coherent sources on the same wavefront after the wavefront has been distorted by some obstacle

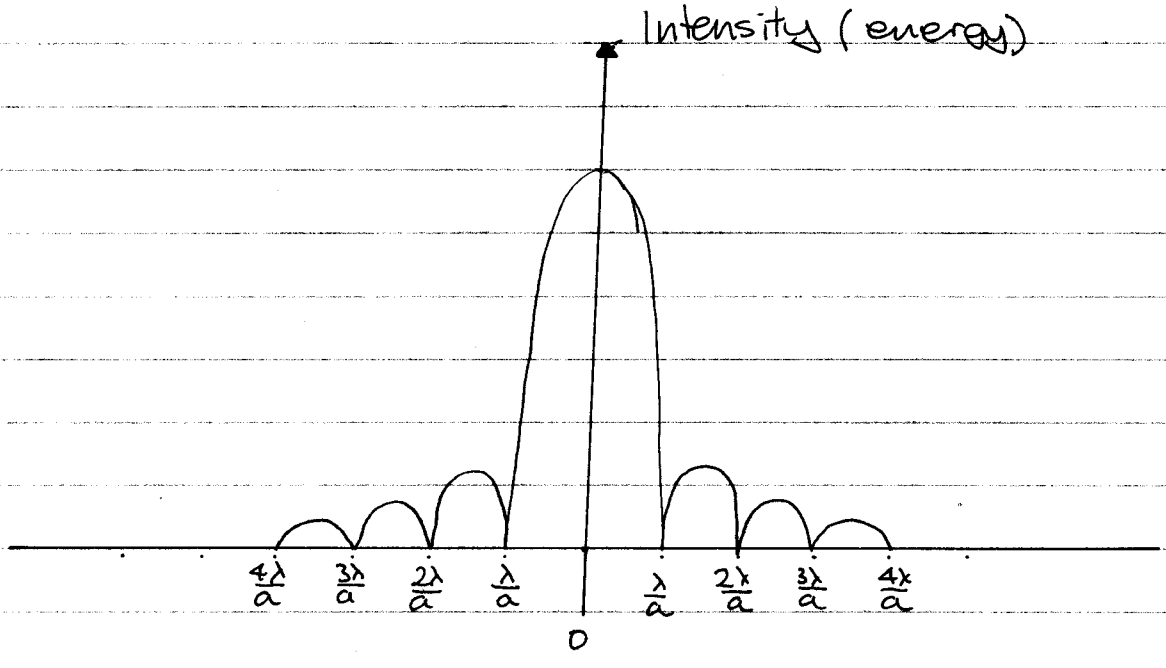
or: Diffraction is the passage of light into a Geometrical Shadow Region.

We can investigate Diffraction at a single slit.

1. A Broad Slit. – Fresnel Diffraction
Not much diffraction except at edges the path broadens out.

2. Medium Slit
A semi circle of light produced.

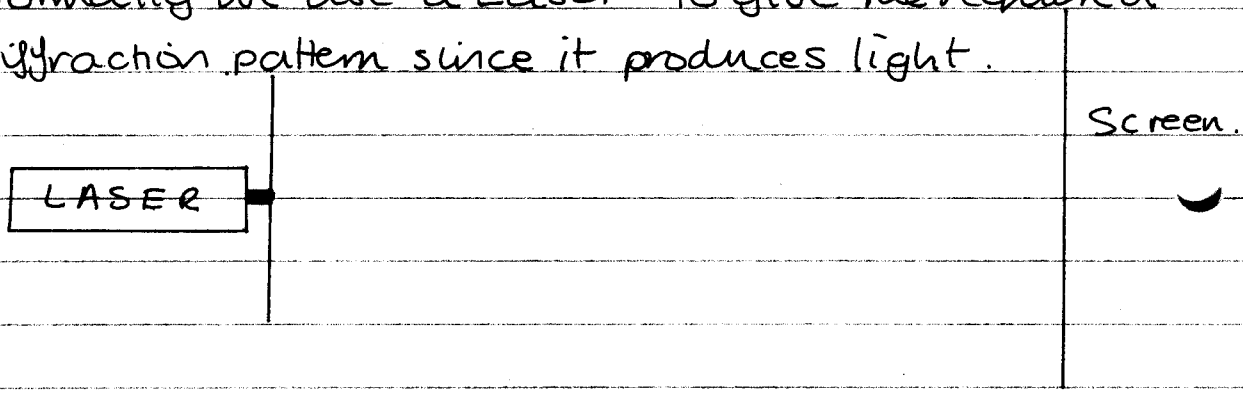
3 Small slit. – Fraunhoffer Diffraction
Here a Fraunhoffer diffraction pattern appears. This is of the general form as below which will be proved:



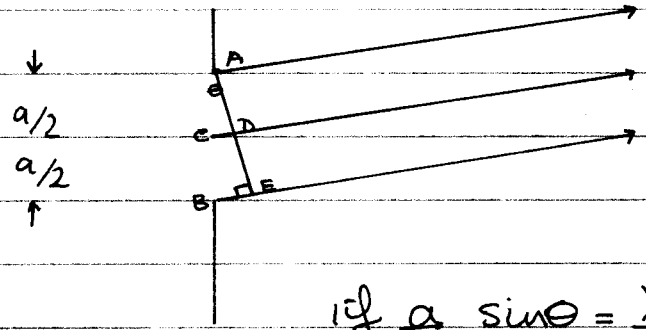
Single Slit Fraunhoffer Diffraction.

To produce this we shine a coherent light source at a slit. Because it is coherent all successive wavefronts have same Phase and Frequency.

Normally we use a Laser to give the required Diffraction pattern since it produces light.



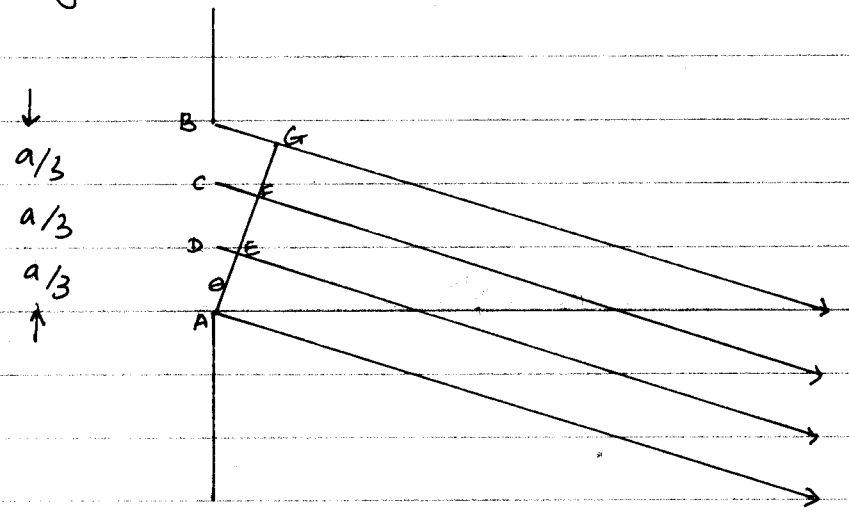
The screen becomes as shown on previous page. We can show these maxima/minima as follows:
We divide the slit into 2 equal parts



if $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ $\therefore \sin\theta = \frac{\lambda}{a}$

then all light from AC cancels light from CB.
Thus at the point where the 2 halfs converge we get a minimum at $\sin^{-1}\frac{\lambda}{a}$

We then divide the strip into 3 equal beams as follows:



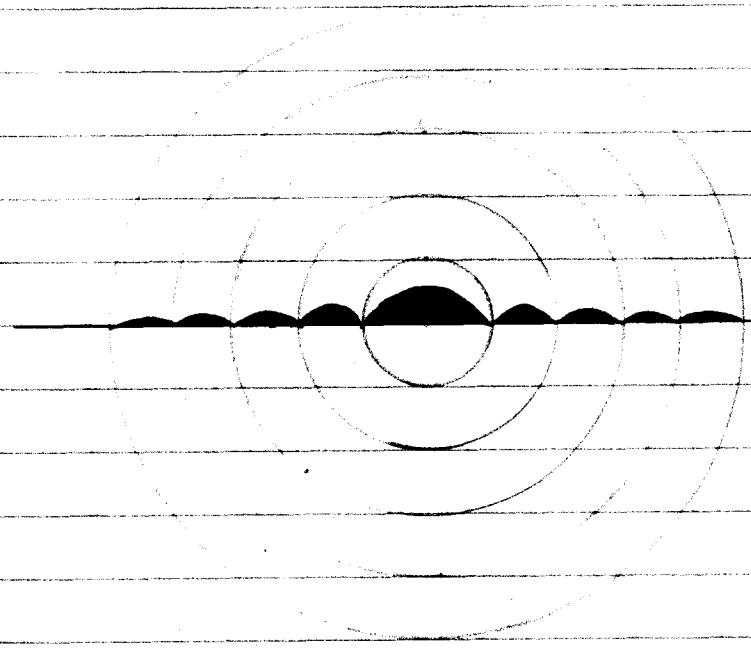
Waves set out from the slit AB all in phase. Considering waves at an angle θ if the path difference is $\frac{\lambda}{2}$ we will get ~~destructive~~ constructive supperposition i.e when $\frac{a}{3}\sin\theta = \frac{\lambda}{2}$

i.e $\sin\theta = \frac{3\lambda}{2a}$

We can further split the screen into 4, 5, -n parts ($n \to \infty$). For even n we get zero intensity due to destructive superposition. For odd n we get maximum intensity due to constructive superposition.

Although Fraunhoffer Diffraction normally occurs with a single slit we can use a a single circular hole.

P.T.O



Circular pattern obtained with a hole

The First Maxima is at $\frac{1.22\lambda}{d}$

λ = wavelength of light applied
d = diameter of hole.

Angular radius $\theta = \frac{1.22\lambda}{d}$.

LOWER SIXTH PHYSICS

DIFFRACTION

1 A parallel monochromatic beam of light of wavelength 600 nm falls on a circular lens of diameter 50 mm and focal length 1.0 m. Calculate the diameter of the first dark ring in the "point image." Use your answer to decide whether the naked eye could detect the diffraction effects. (The closest distance away of an object for distinct vision is 25 cm).

2, using $\theta = \frac{1.22\lambda}{D}$, where θ is the angle of resolution or angle between centre and first dark ring, and D is diameter of aperture:-

a) What is the value of θ for a human eye of pupil diameter 4 mm using light of wavelength 600 nm

b, The Mount Palomar Telescope can just resolve to 0.1 μ radian for blue light (400nm) estimate its objective diameter

c, Jodrell Bank radio telescope has a diameter of 70 m. What is the greatest wavelength it can receive so that it can separate objects subtending an angle of 3.7 m radian?

AMR.

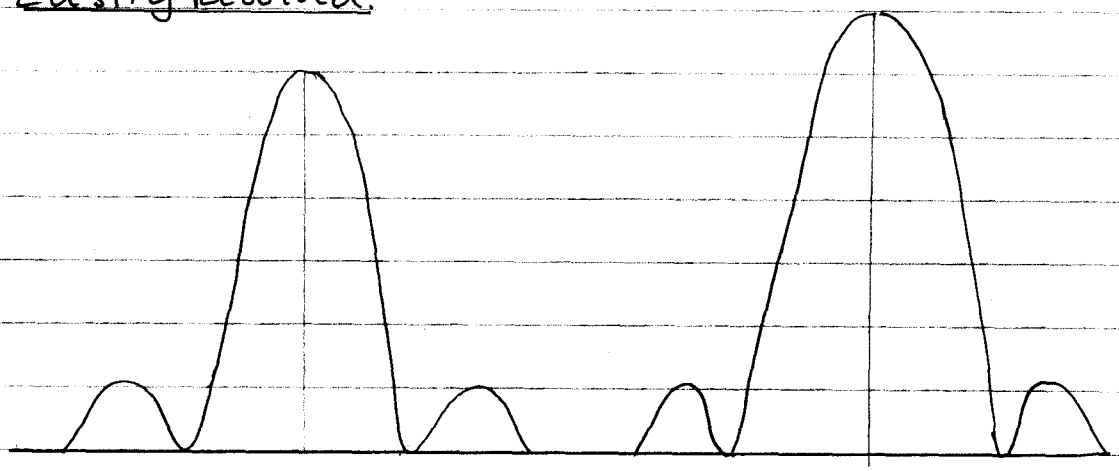
Resolving power.

When we look at any object we see a diffraction pattern for that object due to our own optical system.

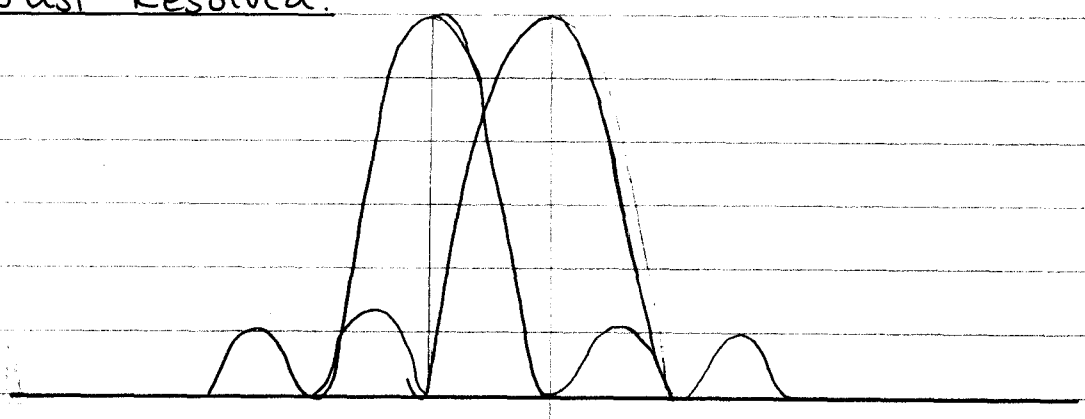
Thus if we look at two distant objects, lights for example they may appear together.

We say 2 objects are resolved if the central maximum of one objects diffraction pattern is directly below the first minimum of the other, or better.

Easily Resolved.



Just Resolved.

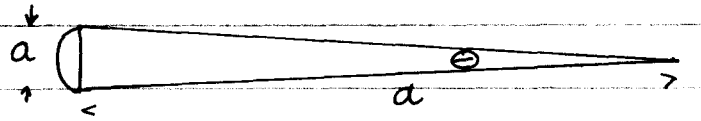


Resolving power is the ability of an optical system to seperate to form seperate images of objects that are very close together and so reveal detail.

1. Resolving Power of the Eye.

The smallest angle θ which two points can subtend at the eye and still be seen as seperate is taken as a measure of the resolving power of the eye.

If they are not seperate it is an indication that diffraction has occured by the pupil.



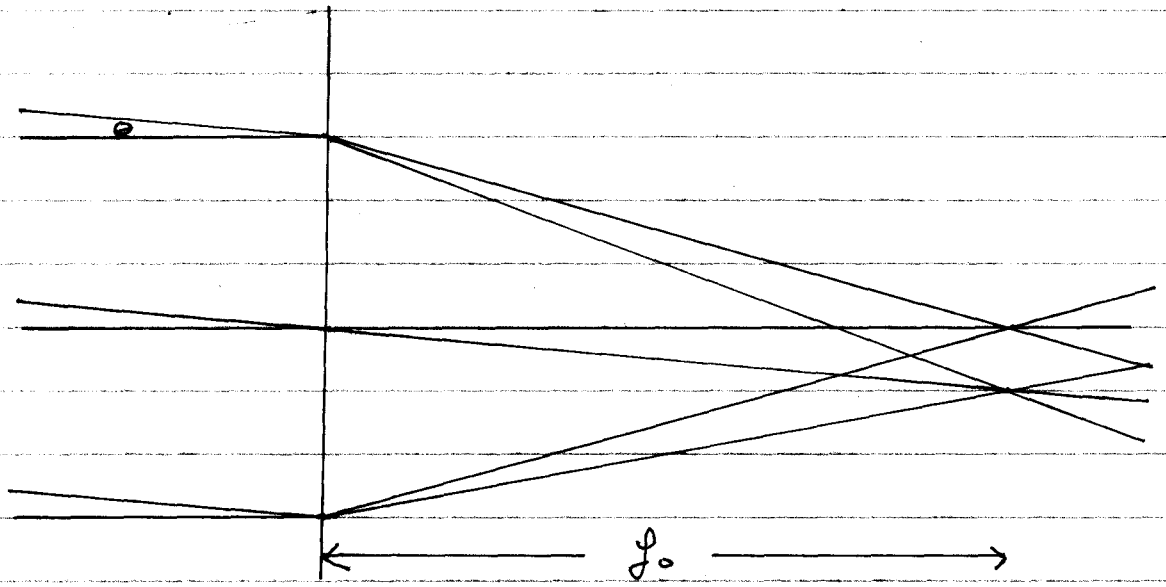
$\theta = \frac{a}{d}$. The smaller θ the greater the resolving power.

The resolving power of the eye with a small pupil is such that θ is about 10^{-4} rad.

e.g we can resolve 2 points 0·1mm apart 25cm from the eye.

Resolving Power

2. Resolving Power of a Telescope



Objective lens.

The above diagram shows the diffraction pattern created by a lens and 2 distant objects

When a lens forms an image of a small object it acts as a circular apeture, then the resultant image formed in the focal plane of the objective represents diffraction patterns. This is because the light has been restricted by the objective lens.

If θ is the angle between the 2 distant objects that must be resolved

Two objects are resolved for θ rads $= \frac{1.22\lambda}{d}$ apart.

Increasing the Resolving power.

Resolving power $= \frac{\lambda}{d} = R$

d = slit width or lens diameter
λ = wavelength of incident light

To increase the resolving power we must decrease the wavelength of light used or increase the diameter of the lens.

It is more common to use short wavelength light such as ultraviolet light to viewscreen. In the electron microscope moving electrons behave as waves but having wavelengths 10^5 that of light the resolving power is much better i.e R decreases.

The resolving power of an instrument limits its useful magnifying power.

Limit of max useful magnifying power $=$ $\frac{\text{Resolving power of Eye}}{\text{Resolving power of telescope}}$

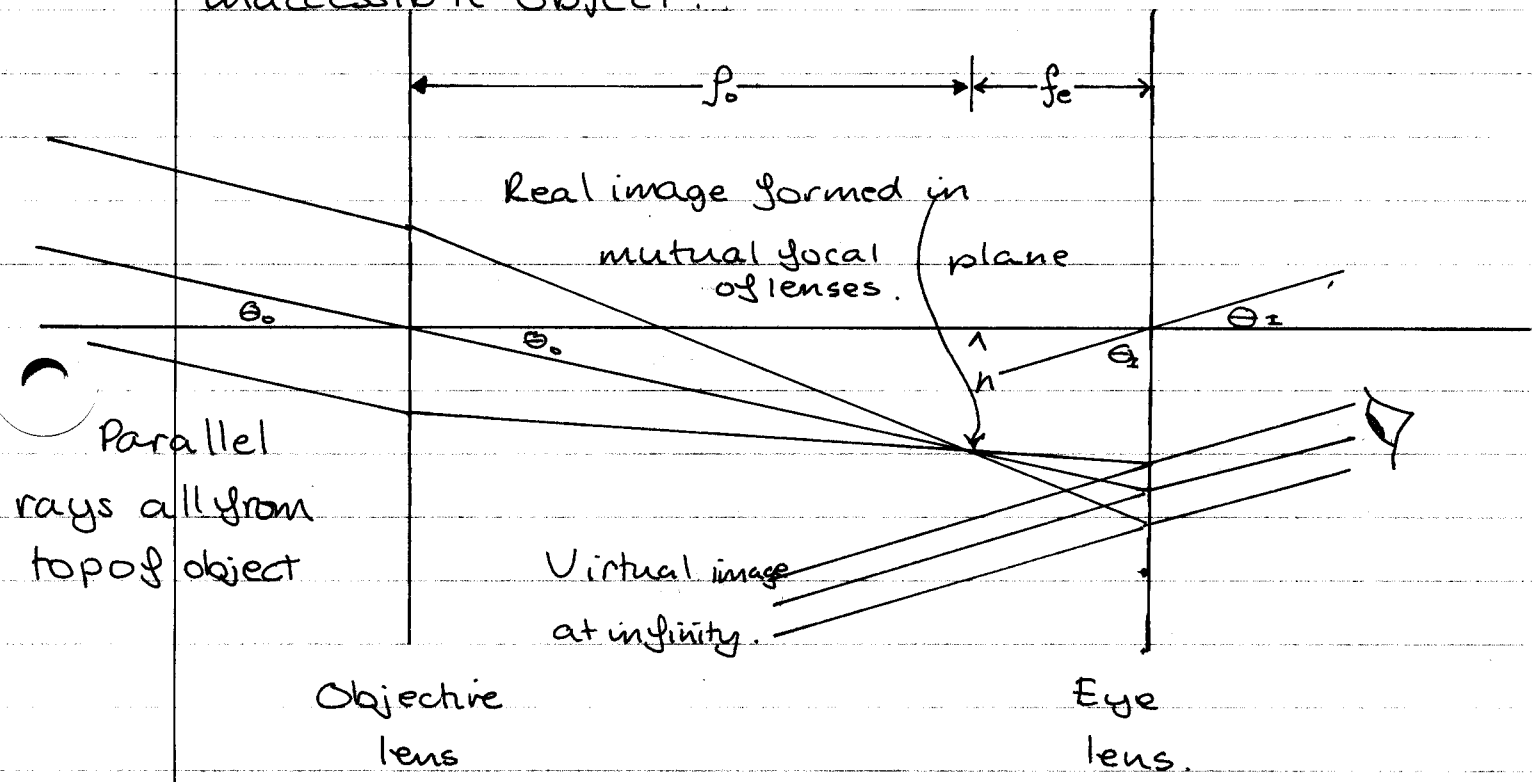
e.g Large Astronomical telescope lens $\cdot$ $d = 5\,m$
Resolving power $= 3\times10^{-4}/5 = 10^{-7}$ for telescope
Useful mag power $= 3\times10^{-4}/10^{-7} = 3\times10^{+3}$

Electron microscope RPower $= 10^{-12}$ rad
Useful Magnifying power $= 3\times10^{-4}/10^{-12} = 3\times10^{+8}$

Any further magnification beyond this limit increases picture size but not resolution.

The Refracting Telescope

The aim of this instrument is to produce an enlarged retinal image of a distant inaccessible object.



Rays from the top of the object subtend an angle θ_o with rays from the bottom. Rays from any one point on the distant object are effectively parallel at the objective lens, so the objective lens forms a diminished real image in its focal plane. The eye lens is used as a magnifying glass, and the final image is formed at infinity when the instrument is in normal adjustment.

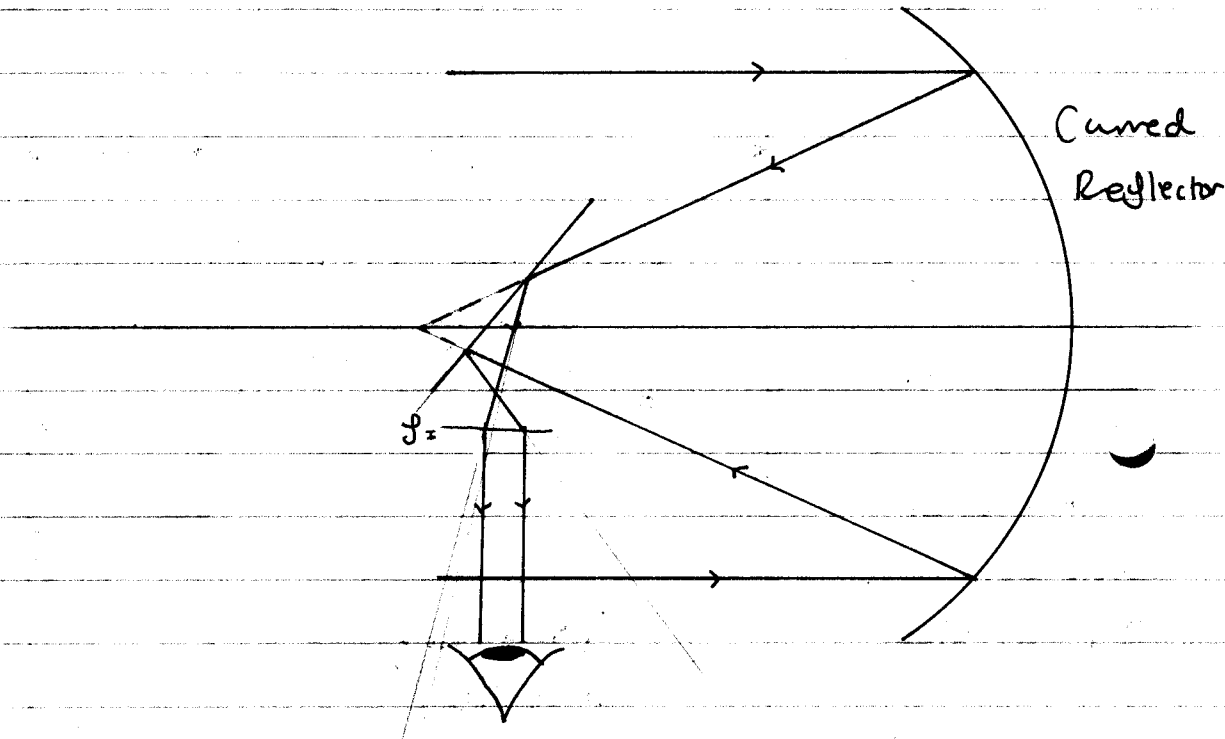
Magnification $M = \dfrac{\theta_I}{\theta_o} = \dfrac{h/f_e}{h/f_o} = \dfrac{f_o}{f_e}$

(a) To make M large we want a large f_o and small f_e

(b) Length of telescope tube $= f_o + f_e$

(c) The resolving power depends on diameter + quality lens

The Newtonian Telescope.

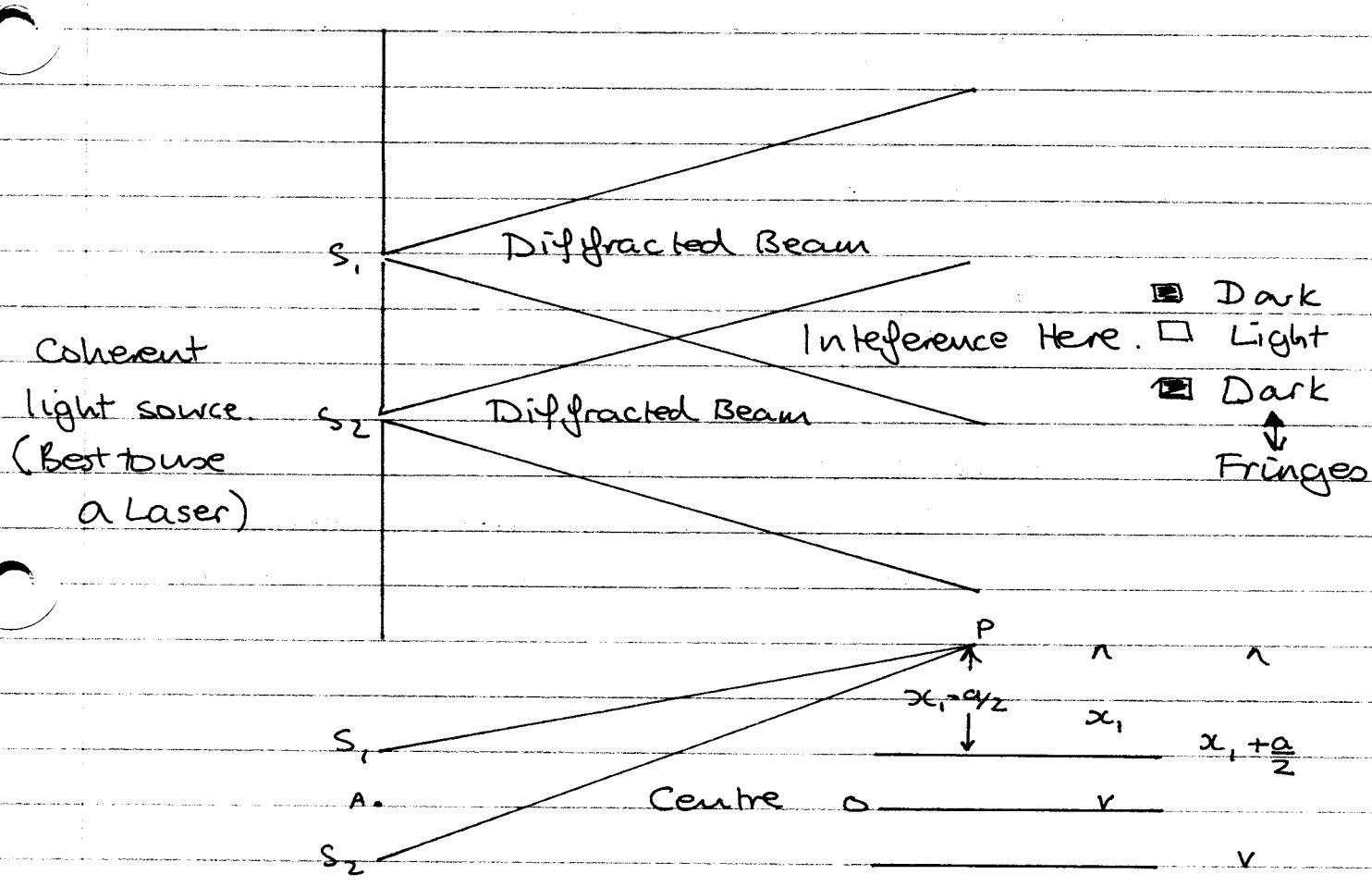


Parallel rays from a distant point are reflected by the objective and then at a small plane mirror to form a real image I, which is viewed through an eyelens.

Inteference.

Inteference occurs when two coherent light sources superpose. The wavelength used must be small to discount diffraction effects.

Illustrated via Youngs Double Slit.



$AO = d$, $\quad S_2P^2 = d^2 + (x_1 + \frac{a}{2})^2$

$S_1P^2 = d^2 + (x_1 - \frac{a}{2})^2$

$S_2P^2 - S_1P = 2ax_1 = (S_2P - S_1P)(S_2P + S_1P)$

$(S_2P + S_1P) \approx 2d$ Thus

$$(S_2P - S_1P) = \frac{2ax_1}{2d} = \frac{ax_1}{d}$$

Thus for first bright fringe $\lambda = \frac{ax_1}{d}$

For the nth fringe $n\lambda = \frac{ax_1}{d}$

If (n+1)th bright fringe has fringe distance x_2

$$\therefore (n+1)\lambda - n\lambda = \frac{a}{2}(x_2 - x_1)$$

$x_2 - x_1$ = distance between dark or light fringes (this is constant).

Let $x_2 - x_1 = y$.

$$\lambda = \frac{ay}{d}$$

a = slit separation
y = distance between like fringes.
d = distance between screen and slit.

Interference.

Measurement of λ

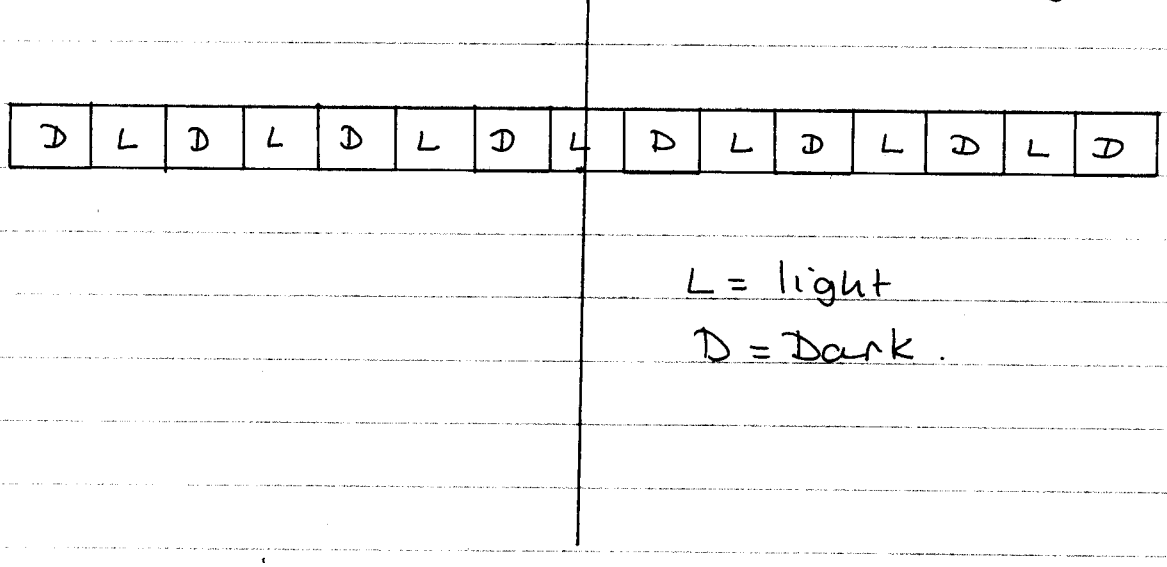
The experiment is set up as in previous diagram with a lazer light source putting light into 2 slits to produce an interference pattern.

The fringes are observed using a travelling microscope in a blacked out room using a cross wire.

The average fringe spacing y is found by measuring across as many fringes as possible with a travelling eyepiece. A metre rule is used to measure d then a. Hence a value of λ is calculated.

$$\lambda = \frac{ay}{d}$$

Typical values: 4×10^{-7}m violet light
7×10^{-7}m red light.



Notes on Youngs Experiment.

1. The overlapping beams which interfere are produced by S_1, S_2. The fringes produced are called non localised fringes.

2. Point sources give the same fring pattern but not as clearly.

3. The number of fringes obtained depends upon the amount of diffraction occuring at the slits and the width of the slits.

4. Usually only a small number of fringes (about 10) are normally seen before the intensity falls to zero.

5. If we use white monochromatic light as the source fever fringes are seen, but all the fringes except centre have a colour to them.

The Spectrometer.

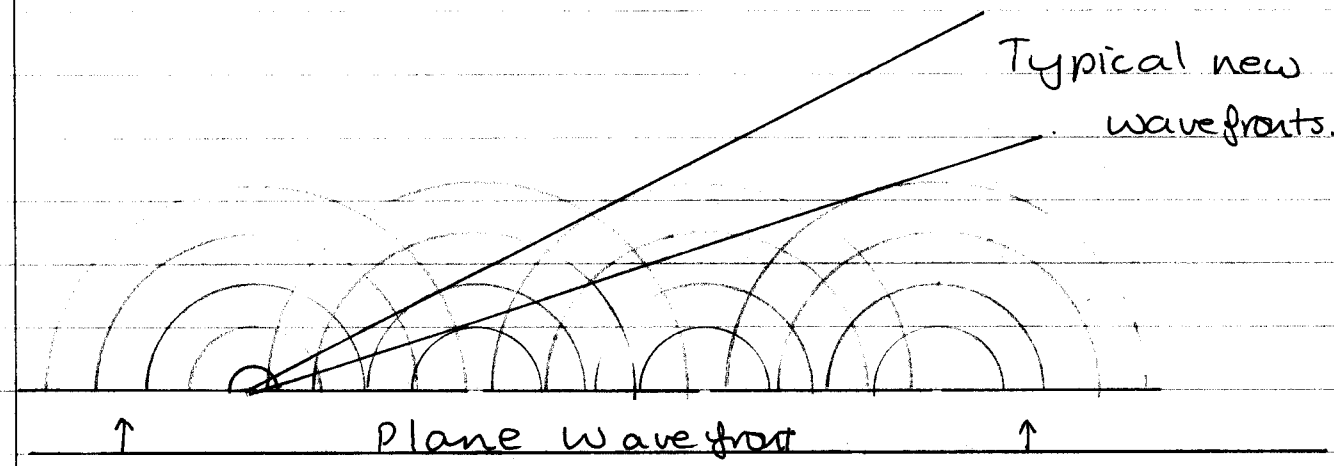
Use, Details of P246 EPP

Only revise if have time, look at freq occurence in past papers.

Diffraction Grating

An optical diffraction grating consists of a large number of fine equidistant parallel lines ruled onto a transparent plate (such as glass) so that light can pass between the lines but not through them.

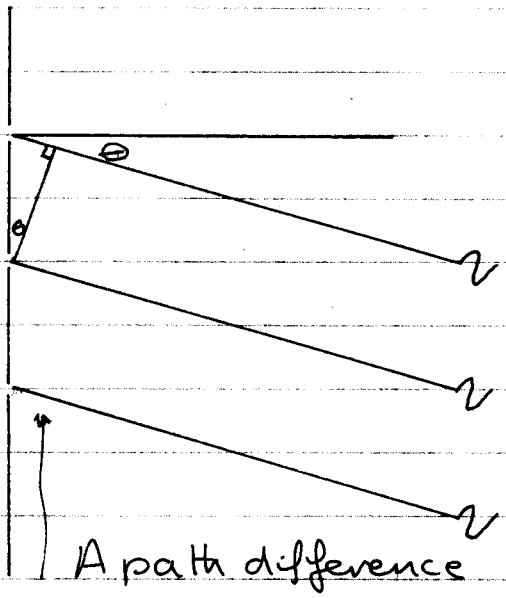
If the gap if sufficiently narrow, diffraction causes it to act as a centre of secondary disturbance. Along a few specified directions (NOT Points) these secondary wavelets combine to produce a continuos wavefront.



Along other directions the superposition is destructive and no wavefront exists

The superposition is constructive if the phase difference from sucessive slits at an angle θ is a multiple of λ rads i.e

Waves set out from slit in phase



All these waves from seperate slits are in phase at angle θ° to give constructive interference.

of exactly 1 is introduced for n of waves from adjacent slits at angle θ°

Constructive interference if $d \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{d}.$$

n = order

λ = wavelength of light

d = grating spacing.

The number of orders that can be obtained is dependent upon the wavelength of incident light and grating spacing.

Spectra.

White light is a mixture of all visible light from Red ($7.5 \times 10^{-7} m = \lambda$) and violet ($\lambda = 4 \times 10^{-7} m$).

When white light is shone onto a diffraction grating spectra are seen.

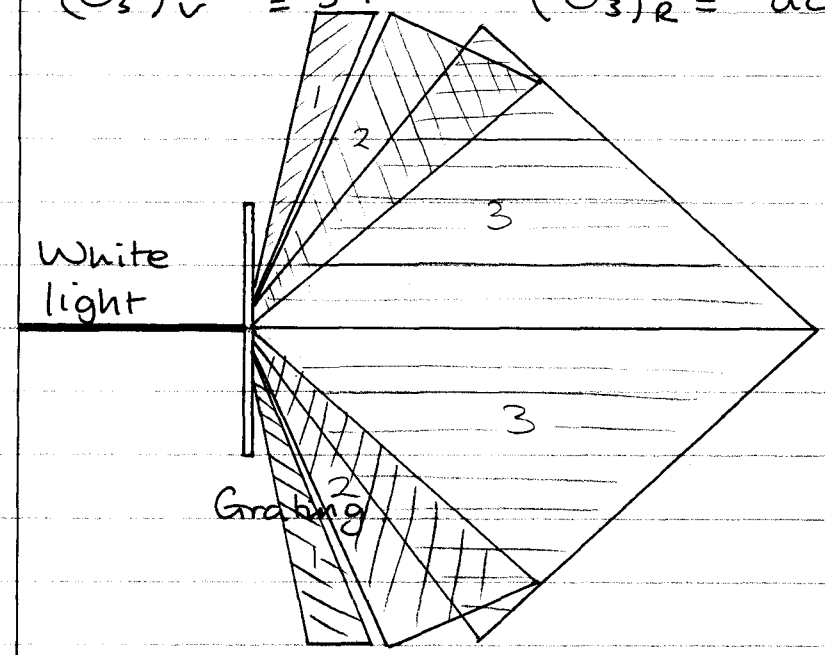
We can calculate this for a ~~typical~~ typical example

Using a grating spacing of $s = 2 \times 10^{-6} m$

$(\theta_1)_V = 12°$ $(\theta_1)_R = 22°$

$(\theta_2)_V = 24°$ $(\theta_2)_R = 49°$

$(\theta_3)_V = 37°$ $(\theta_3)_R =$ does not exist ∴



Crystal Structure.

Atomic/Molecular mass.

The Atomic or Molecular mass of an atom is numerically equal to the mass of 1 mole of those atoms in grams. This is taken with respect to 1 mole of carbon which weighs exactly 12 grams.

Avogadro Constant

This is the number of elementary units in 1 mole of the compound.

Interatomic Bonds

These are divided into Ionic, Covalent, Metallic. Metallic bonds have loosely held outer electrons that are easily lost. So in a metal many free electrons exist all moving randomly, all the atoms sharing all the electrons. The strong attraction between the ions and electrons constitutes the metallic bond.

States of Matter

1. Solids

In a solid the interatomic force i.e that between neighbouring atoms is zero. The equilibrium seperation which constitutes the solid is altered then repulsion or attraction occurs. In the solid state atoms vibrate about their equilibrium positions, alternately attracting and repelling one another.

2. Liquids.

If the temperature in a solid is increased sufficiently the larger atomic vibration may be so pronounced as to allow individual atoms to escape neighbourly attraction. A liquid is thus formed.

Although the forces between the molecules are not large enough to support a definate shape they do give rise to properties such as surface tension, viscosity, latent heat of vapourisation.

3. Gases.

In a gas/vapour state atoms and molecules move randomly with high speeds in large volumes far apart from one all another. Mollecular interaction occurs on collision only when a large repulsive force is generated.

Atomic / Molecular Mass

The Atomic or Molecular mass of an atom is numerically equal to the mass of 1 mole of those atoms in grams. This is taken with respect to 1 mole of Carbon which is said to weigh exactly 12 grams.

Avogadro Constant

The Avogadro constant is the number of molecules in the Molecular mass of a compound (one mole of).

Interatomic Bonds.

These are Divided into Ionic, Covalent and Metallic. Ionic and Covalent are self Explanitory.

Metallic Bonds:

Metal atoms have 1 or 2 outer electrons that are loosely held and easily lost. In a metal there are many free electrons which move randomly, all the atoms sharing all the electrons. The strong attraction between the ions and electrons constitutes the metallic bond.

States of Matter

a Solids.

In a solid the interatomic force i.e the force between atoms is zero. The equilibrium seperation which constitutes the solid is altered is the seperation is altered. Further apart and attraction occurs. Closer together and there is repulsion. In the solid form atoms vibrate about their equilibrium positions alternatively attracting + repelling one another. This position is a mostly locked position and so solids have shapes of appreciable stiffness.

(b) Liquids.
If the temp of a solid is sufficiently increased the larger atomic vibration may be so pronounced that individual atoms can escape their neighbourly attraction. If this happens a solid melts and a liquid forms.

Although the forces between the molecules in the liquid are not enough to sustain a definate shape they do support the properties of surface tension, viscosity and latent heat of vaporisation.

(c) Gases
In a gas/vapour atoms and molecules move randomly with high speeds in large spaces often far apart from each other. Molecular interaction occurs in collisions only when a large repulsive force is generated.

Types of Solids
There are two main types of solids.

Crystalline.
This is the structure of most solids (all metals) and many minerals. Here the essential crystal structure is that the arrangement of atoms, ions, or molecules repeats itself regularly many times i.e there is a long range order.

Amorphous.
Here the particles are arranged in a disordered manner showing no long range order. Example solid is glass.

Strengthning of Materials.

The weakness of a pure metal can be attributed to a moderate number of easily movable dislocations in the crystal structure of a metal. 3 ways are present; to strengthen:

(a) Foreign atoms.
In steel for example carbon atoms are introduced to the steel to oppose the disslocation motion so increasing strength and stiffness.

(b) other dislocations.
If the metal is submitted to further stress, dislocations are created to a point where they obstruct one another.

(c) Grain Boundries.
Most metals are Polycrystalline that is they consist of many small crystals or grains at different angles to one another. The boundary between two grains is imperfect acting as an obstacle to a dislocation movement so giving the material more strength.

Cracks + Fracture

(a) Brittle Fracture.
Occurs by the rapid propagation of a crack. In a brittle material cracks spread rapidly under tension but slowly if at all under compression. This explains the strength of prestressed concreate or glass.

(b) Ductile Fracture
This follows slow crack propagation. If after thining under plastic elongation the material develops a waist or neck cavities form which join up into a crack which travels the surface of the specimen.

M. Bennett.

Stress + Strain.

Strength tells us the ability of a material to withstand an applied force upon it.

Stress.

Stress is the force acting on unit cross section area. Thus for a force F Area A -.

Stress = F/A [Pascals] $Pa = Nm^{-2}$

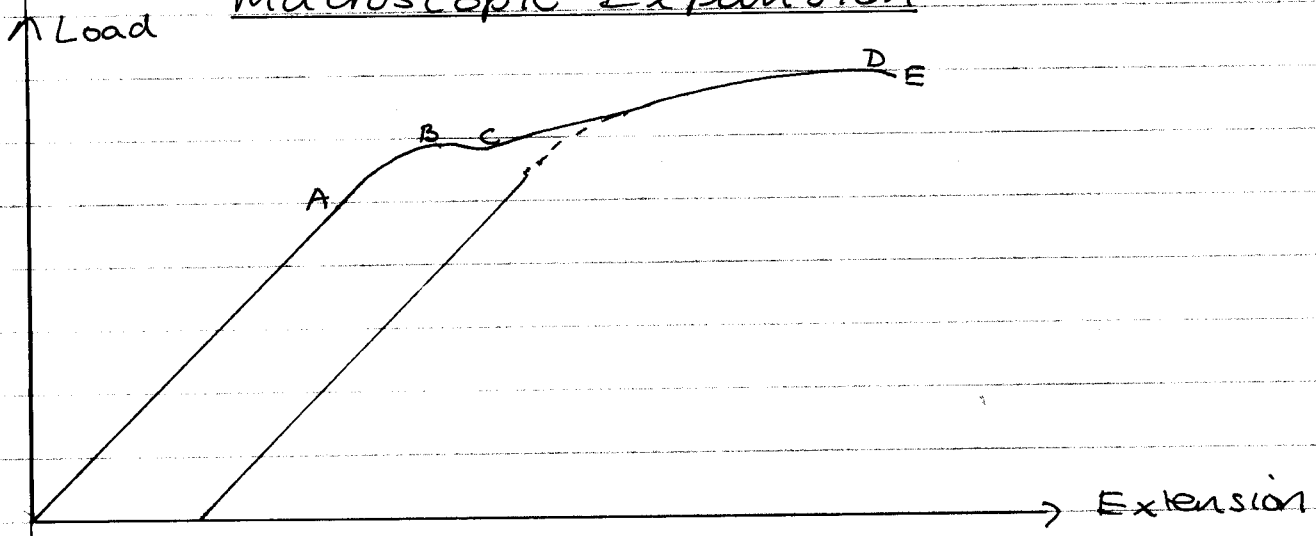
Strain.

Strain is the extension of unit length

For e = extension l = original length

$$\text{Strain} = \frac{e}{l}$$

Macroscopic Expansion



O - A	Elastic Deformation
A	Limit of Proportionality
B	Elastic Limit
C	Yield Point
C - D	Plastic Deformation
D - E	Necking Occurs
E	Wire Breaks.

Elastic Deformation - Extension proportional to the applied load.

Limit of Proportionality.
Beyond this point Load is not proportional to extension.

Elastic Limit
This is the largest length possible for unloading to give a return along the loading line.

Plastic Deformation
The recovery of the original shape is no longer possible and wire will return ~~to~~ along the blue unloading line as shown.

Necking.

Here the wire thins via the slipping of the atomic planes at a weak point.

Breaks.
Prolonged necking causes wire to break.

Youngs Modulus

$$\text{Stress} = \frac{\text{Force Applied}}{\text{Area over which it acts.}}$$

Strain = Fractional increase in length.

$$E = \text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}} \quad \text{(Tensile)}$$

$$E = \frac{FL}{Ae} \quad \text{(Pascals)}$$

If a force F is provided by a mass M hanging on the end of the wire then:

$$M = \frac{EAe}{Lg}$$

Energy Stored in a Stretched Wire

Force F needed to extend a wire by x given by

$$F = \frac{EA}{L} x$$

Work done to stretch it from x_1 to x_2 $(x_2 > x_1)$

$$= \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \frac{EA}{L} x$$

$$= \frac{EA}{L}\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{EA}{2L}(x_2 + x_1)(x_2 - x_1)$$

$$= \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume}.$$

$$\frac{\text{Work}}{\text{Volume}} = \frac{1}{2} \text{ Stress} \times \text{Strain}.$$

E = Youngs Modulus
A = Area
L = Length
x = extension.

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CSA = Cross sectional Area.

①

(b) Copper wire ~~of Area~~ 1mm x 1mm. Stretched by 40N Calc tensile stress. $= \frac{\text{Force}}{\text{Area}} = \frac{40}{(\frac{1}{1000})^2} = 4 \times 10^7$ Pa ✓

2

(c). Break stress steel $= 1 \times 10^9$ Pa will wire of area 4×10^{-4} cm^2 break when 10kg mass hung from it.

Stress $= \frac{\text{Force}}{\text{Area}} = \frac{10 \times 10}{4 \times 10^{-4} / (100)^2} = 2.5 \times 10^9$ Pa ✓

Stress wire experiences > than Breaking stress so wire breaks.

2

(d). 6cm rubber stretched to 9cm.

Tensile Strain $= \frac{\text{Extension}}{\text{O. Length}} = \frac{(9-6)\text{ cm}}{6\text{ cm}} = \frac{1}{2}$ ✓

Tensile strain $= \frac{1}{2}$ ~~(ratio)~~

50% (percentage). ✓

2

(e). Wire 2m original suffers 0.001 strain

Calc stretched length. = S.

so $0.001 = \frac{S-2}{2}$ so $S = 2 + 0.002 = 2.002$m

Stretched length = 2.002m.

2

(a). Stresses + strains are used instead of forces and extensions because they provide a relative measure of force/area and extension/length ratios. Thus two strains or stresses can be compared to one another and information deduced about the relative strain/stress properties relative to another. This comparison cannot be made with forces, extensions. /

2. Youngs modulus steel > than that for brass.

$$\text{Extension} = \frac{\text{Force} \times \text{Length}}{\text{Area} \times \text{Youngs Modulus}}$$

The greater extension for an applied load would be given by Brass This can be deduced from the above equation.

Stiffness

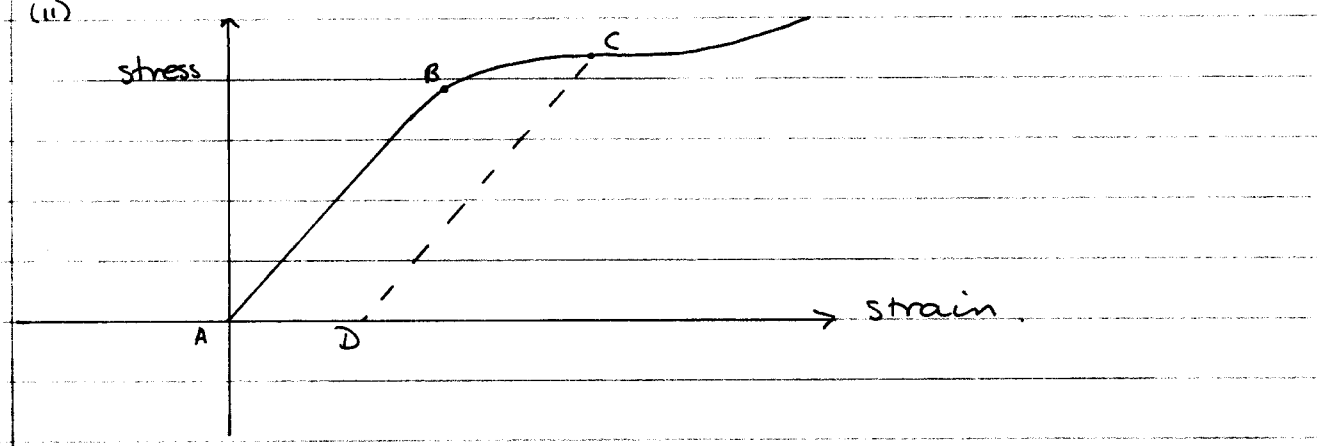
Stiffness is a materials resistance or ability to resist distortion in shape or size.

Thus Brass is not as stiff as Steel for it is more easily extended than Steel when a load is applied to it. 2

(b). Under elastic strain the material will return to its original length when the stress is removed and / none of the extension remains. This is assuming a zero elastic hysteresis. If however there is some Elastic Hysterisis then the unloading Strain will lag behind the loading strain.

M. W. B

2b (ii)



IF A is taken to be the origin and B the yield Point then: Under the removal of a plastic strain at C say, recovery is no longer complete and the specimen recovers along CD which is // to AB. After reaching D no further recovery occurs. ✓ 2

(c) 2.5m brass wire CSA = $1.0 \times 10^{-3} cm^2$ stretched 1 mm by 0.40 kg.

Calc youngs modulus $E = \frac{FL}{Ae}$ (F = force, L = original length, A = CSA, e = extension)

$$E = \frac{0.40 \times 10 \times 2.5}{1.0 \times 10^{-3} \times (\frac{1}{100})^2 \times 1/1000} = 1 \times 10^{11} Pa$$ ✓

Youngs modulus = $1 \times 10^{11} Pa$

% strain suffered = (Extension / Orig Length) * 100

$= [(1/1000) / 2.5] * 100 = 0.04\%$

% strain suffered = 0.04 ✓

What force to give 4.0% strain.

0.04 = Extension / 2.5 ∴ Extension = 0.1m

So Force Required = $\frac{\text{Youngs Mod} \times \text{CSArea} \times \text{extension}}{\text{Orig Length}}$

$= \frac{1 \times 10^{11} \times 1 \times 10^{-3} \times (1/100)^2 \times 0.1}{2.5} = 400N.$

Force Required = 400 Newtons. ✓

After a 4% strain the wire would most likely have passed its elastic limit thus the force applied would be considerably less than 400 Newtons to provide a 4% strain.

CJ
10

P144 Nos 1, 2, M. Bennett.

1. A temperature scale is a scale on which different degrees of hotness may be represented. ✓

The scale is established by first choosing a device which possesses a measurable Thermometric property. This is then calibrated against 2 or more known fixed points.

If X is a thermometric property and we let:

X_u value at upper fixed point
X_L value at lower fixed point
N a constant multiplier
T_L scale fixer being chosen temp value at X_L
T_m temperature to be measured at X_r

$$T_m = \frac{X_r - X_u}{X_u - X_L} N + T_L$$ ✓

(ii) Thermometers based on different properties give different values for the same temperature except at the fixed points where they must agree by definition. The reason for the conflicting values stems from the different Thermometric properties which each thermometer uses, none of them responds in the same way and by the same token none are exactly linear. All are correct however according to their own scale.

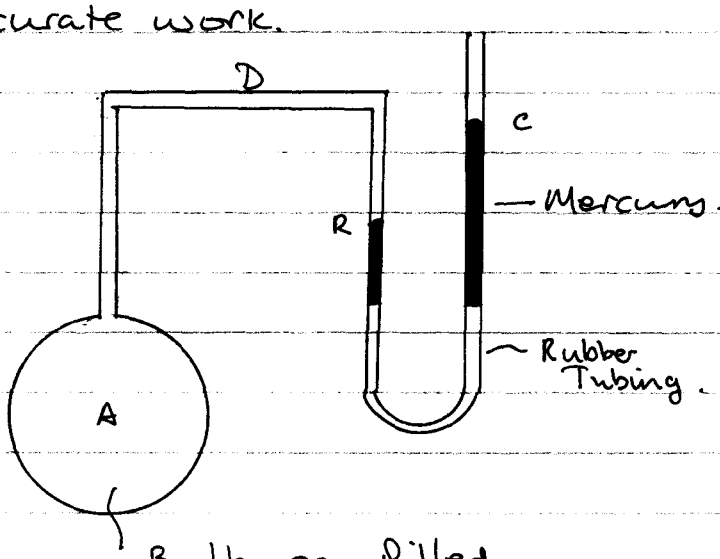
The disagreement between scales has led to a standardisation in which all temperatures are measured called the Thermodynamic scale. The zero of this scale (which is expressed in kelvin) is absolute zero below which temperatures cannot exist. The upper fixed point is given as 273.16 K which is the

Triple point of water.

Thermometers for fast + slow changing temps.

Accurate measurement of slow changing temps use CONSTANT-VOLUME GAS Thermometer.

If the volume of a fixed mass of gas is kept constant then considerable pressure changes accompany temperature ones thus making it suitable for highly accurate work.

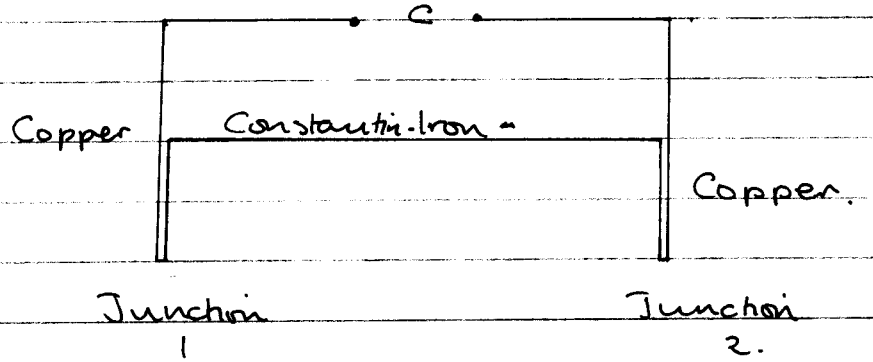


As temperature changes the corresponding change in pressure alters the mercury level. If this level is adjusted to keep R at a constant level for constant volume then we can read off the pressure change C-R. From this knowing 2 levels at 2 fixed points we can calculate any temp measured.

For accurate work corrections are made for unequal temp at D vs A, thermal expansion of A and capillary effects of the mercury surfaces.

For fast changing temps use THERMOCOUPLE Thermometer.

This is ideal to measure fast changing temps as the speed of reading is governed only by the speed of the metal to heat up or cool down. (The metal that of the thermocouple.)



Thermocouple consists of two copper wire connected either side of an Iron wire. Junction one and junction 2 are kept at different temps normally one is the lower fixed point the other a probe. When the Probe is placed in contact with something of different temp an emf is generated which is measured at C. For accurate work C would be a potentiometer or modified Wheatstone Bridge for less accurate readings a Galvinometer can be used.

Today instead of metal thermocouples some semiconductor materials may be employed being more accurate and sensitive than their corresponding metal counterparts. Such a semiconductor would be Iron Disilicide.

P144 PDunc Q2.

2. $$\Theta = \frac{R_\theta - R_0}{R_{100} - R_0} \cdot 100 \quad ^\circ C.$$

Given $R_0 = 2.000\,\Omega$ $R_{100} = 2.760\,\Omega$ $R_\theta = 2.480\,\Omega$.

$$\Theta = \frac{2.480 - 2.000}{2.760 - 2.000} \cdot 100 = 63.158\,^\circ C \text{ (3dp)}.$$

Temp in degrees Centigrade $= 63.158\,^\circ C$ (3dp) *

* [Accuracy to 3 Decimal places as Question is]
Sig figs more appropriate.

(b) Resistance of a wire in $\theta\,^\circ C$ measured on a standard scale is given by

$R_\theta = R_0(1 + A\theta_s + 0.001 A\theta_s^2)$ A = a constant.

Thermometer 50°C on standard scale what on Resistance scale? Let $\theta_s = 50^\circ C$

$$\theta_R = 100\left(\frac{R_{\theta_s} - R_0}{R_{100} - R_0}\right) = \frac{A\theta_s + 0.001 A\theta_s^2}{R_0(1 + 100A + 10A) - R_0}$$

Let $\theta = 50$

$$\theta_R = 100 * \left[\frac{50A + 2.500A}{(100 + 10)A}\right] = \left[\frac{52.5}{110}\right] * 100 = 47.7^\circ C.$$

Temp on Resistance scale $\theta_R = 47.7^\circ C$.

M. Bennett.

Heat

Heat is a form of energy as it is transferred. Energy change may be considered in 2 parts:

1. Work.

Work is the energy transmitted from one system to another by a force moving its point of application

2. Energy (Heat)

Heat is energy transmitted when work is done on a macroscopic scale. The energy flows from one point to another because a temperature gradient exists.

Temperature.

Temperature is defined by the Zeroth law of thermodynamics.

: If two bodies A and B are each in thermal equilibrium with a 3rd body C then they are in equilibrium with each other, and are said to have the same temperature. C is called a thermometer.

Temperature Scales

In order to measure different temperatures comparisons have to be made with certain agreed fixed point temperatures.

We choose

(i) A device with a measureable Thermometric Property i.e it varies with temperature.

(ii) 2 fixed point temperatures.

We measure the Thermometric property at the fixed points and at the temperature to be measured

If Thermometric property has values
X_u at Upper fixed point
X_L at Lower fixed point
X_R at the unknown temperature

If N is some constant, t = constant.
T_R = measured temperature

$$T_R = \frac{X_R - X_L}{X_u - X_L} \times N + t$$

By definition we assume the equation to be linear.

Heat.

It is assumed that the Thermometric property chosen is linear in the range considered and so the Thermometric property must be stated as the same temperature may have different values on different scales.

e.g 60°C on a mercury thermometer.

and 60°C on a mercury thermometer may not equal 60°C on a thermocouple!

Celsius Temperatures.

If Θ is a celsius temperature then:

$$\Theta = \frac{X_R - X_L}{X_U - X_L} \times 100$$

where

X_U = Thermometric property at steam point

X_L = Thermometric property at ice point.

X_R = Thermometric property measured.

A Kelvin temperature refers to the pressure of a constant volume of an ideal gas.

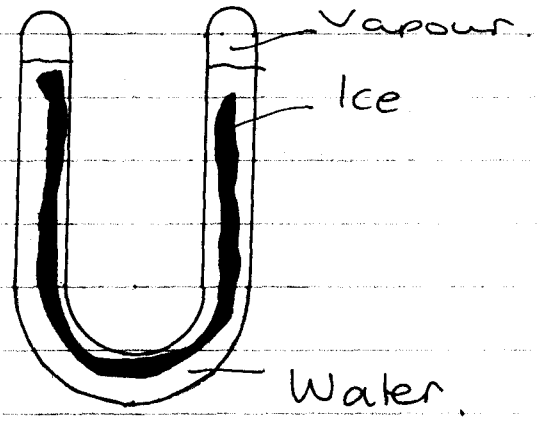
$$T_K = \frac{P_r}{P_{tr}} \times 273.16 \quad \text{Kelvin}$$

P_r = Pressure at required temperature.

P_{tr} = Pressure at the triple point of water.

The Triple Point Cell.

The triple point cell is used to find the pressure P_{tr} at the Triple point.



A U tube open at one end is heated until there is no air in it and then sealed trapping some steam in it.

The cells are usually stored in refridgerator and are at the right temperature which is 0·01°C when ice, water and vapour all coexist.

 M. Bennett.

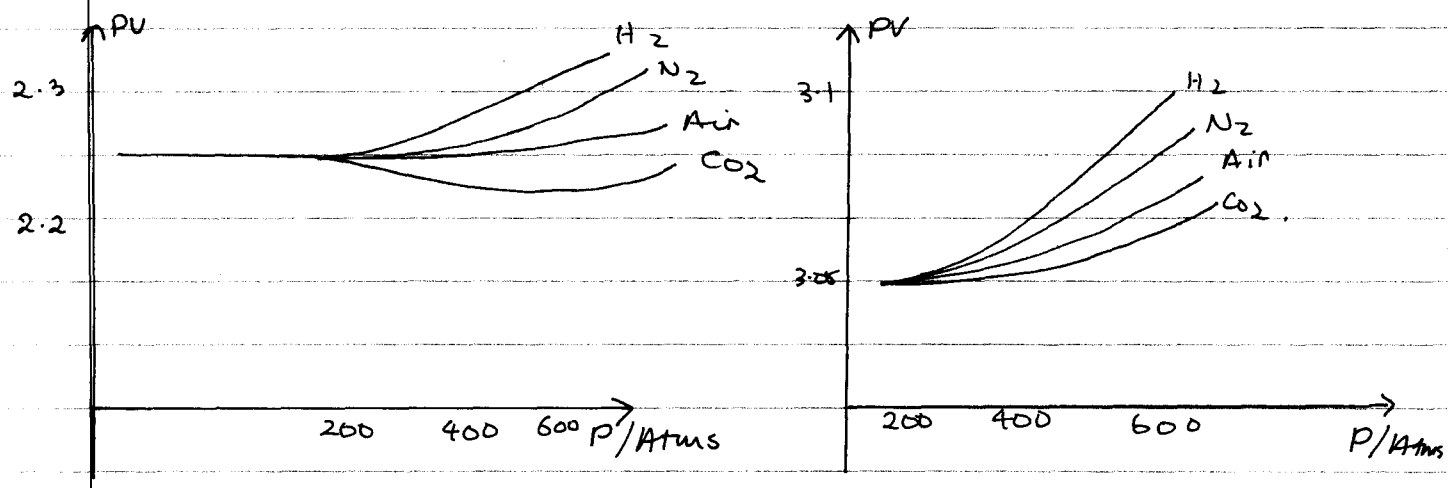
A Scientific Temperature scale.

1. The Kelvin Scale.

There exists an absolute thermodynamic scale proposed by KELVIN. This is based on the theoretical efficiency of a perfectly reversible heat engine. Unfortunately, although this is not dependent on the properties of any particular substance, it is theoretical and cannot be put directly to use.

2 Real Gas Scales.

Although the ideal gas does not exist, it is found in practice that different real gas scales give particularly close agreement over a wide range of temperatures when the gas pressure is low since the gas molecules take up a negligeably small volume and are also so remote that no Intermolecular forces are involved.



At Ice point

At Steam point

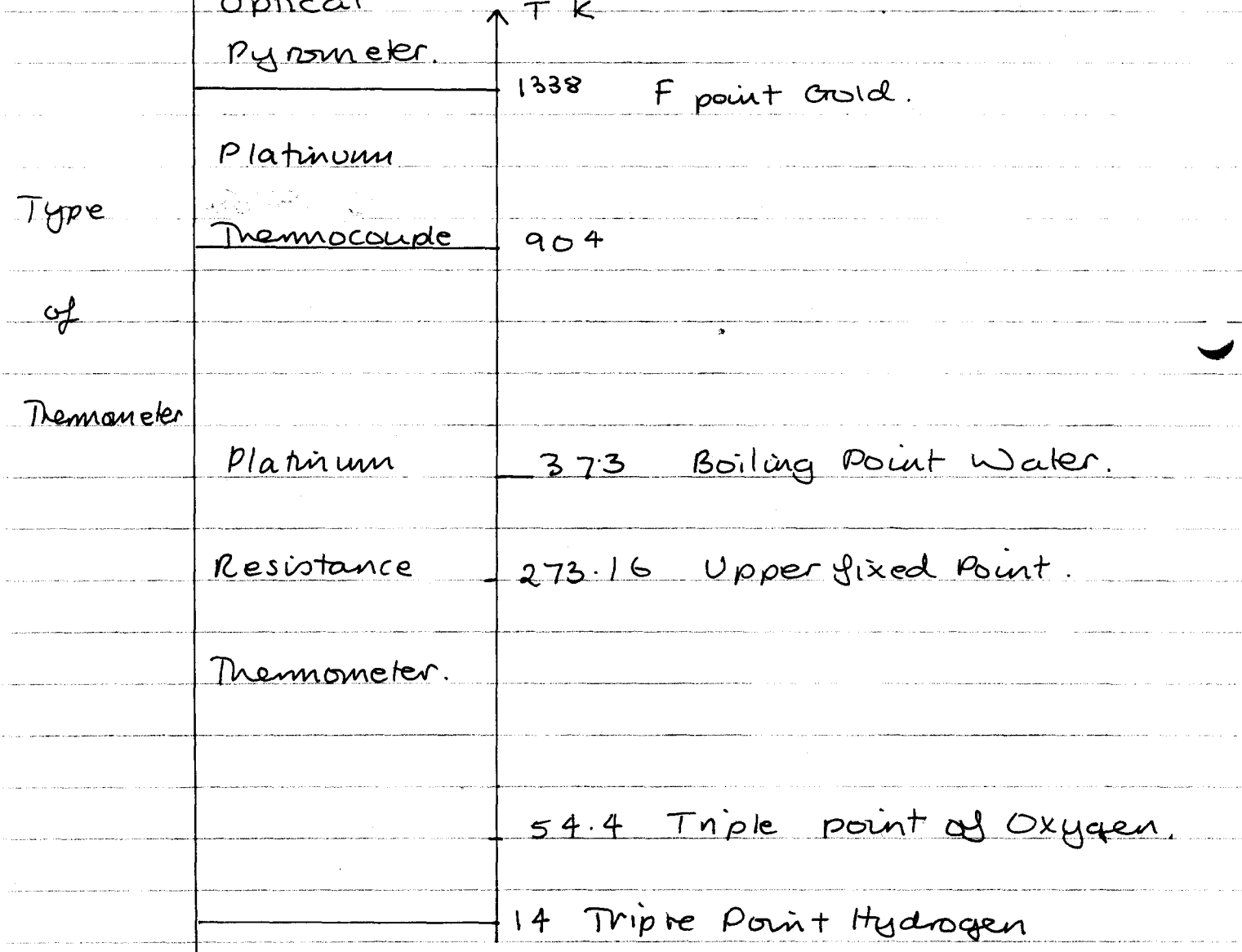
1 Atm = 1×10^5 Pa.

The International Practical Temperature Scale.

The International Practical temperature scale ~~is~~ consists of a number of fixed points together with recomended thermometer for use within several temperature ranges.

Using the standard thermometer temperatures have been assigned to a number of reproducible equilibrium states. They are known as fixed points and available for calibration purposes.

A particular instrument is specified for measuring temperatures within a particular range.



Thermometers.

1. Mercury in Glass Thermometer.

The thermometric property is the apparent linear expansion of mercury in a uniform capillary

$$\Theta = \frac{L_\theta - L_0}{L_{100} - L_0} \cdot 100 \quad ^\circ C$$

Range $-39^\circ C$ to $500^\circ C$.

Very convienient but not a standard.

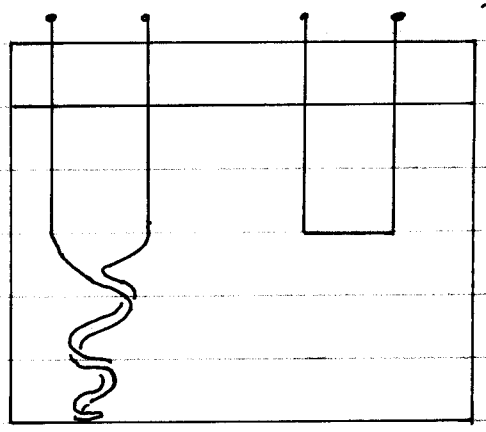
Inaccuracies:

(a) Non uniformity of the bore

(b) Change in the ~~expan~~ shape of the glass

(c) Non uniform mercury expansion

2. Plantinum Resistance Thermometer

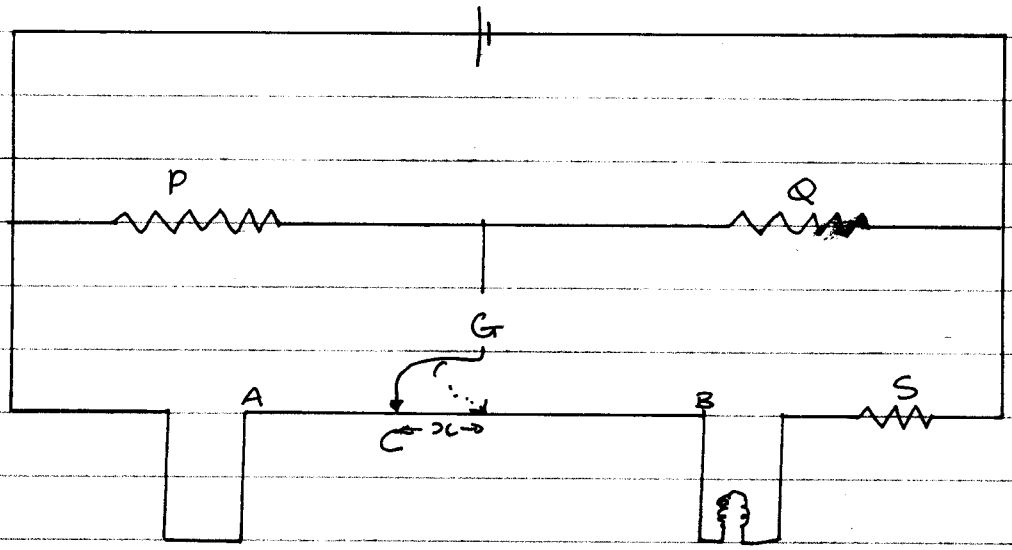


Non inductively wound wire.

Thermometric property is the resistance of the Platinum coil.

Range 83-1400K STD 83-904K.

We attach the previous piece of a equipment to a modified wheatstone Bridge.



Procedure.

(I) Select P accurately to Q

(II) Position the jockey along AB at C so bridge is balanced.

$AC = l_1$ $\quad BC = l_2$

(III) Put the Porcelain container into the enclosure the temperature of which is required. Suppose the jockey has to be moved a disance x to the right to retain the balance.

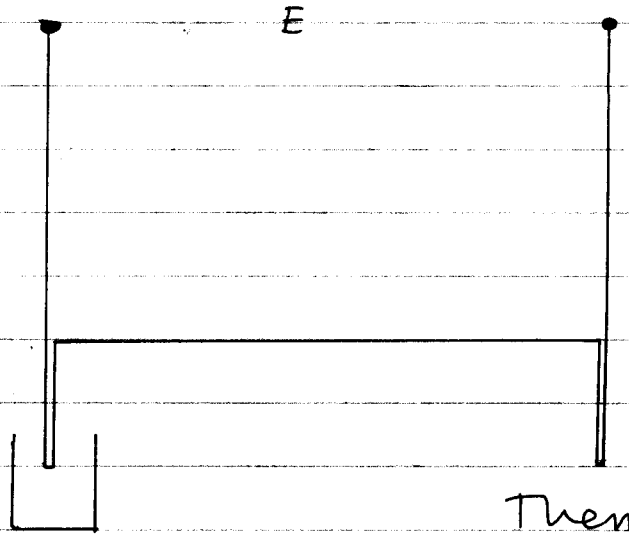
because $P = Q$

$$S + (l_1 + x)r = R_T = (l_2 - x)r$$

Where R_T is the new coil resistance.

If one assumes a parabolic relation between R_T and temperature measured on the ITPS then calibration at three fixed points enables that temperature to be calculated.

The Thermocouple.



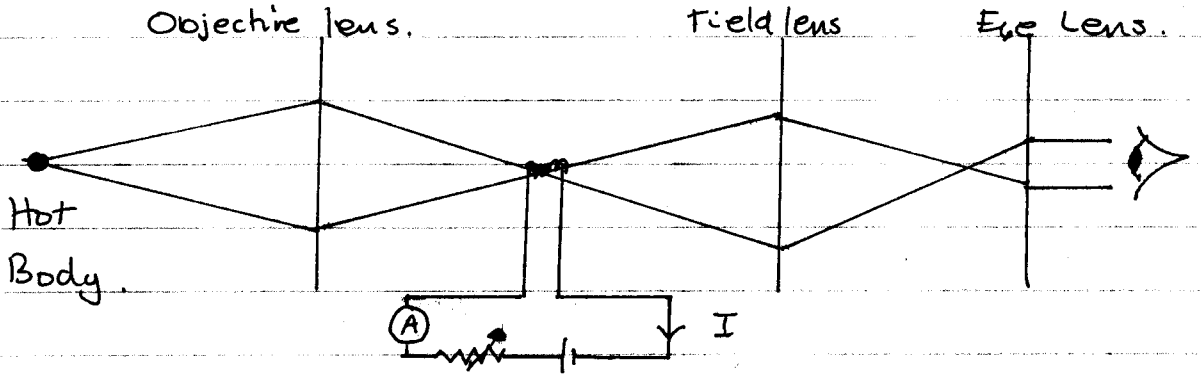
Range 25 - 1750 K STD 90 4 - 1337 K.

The cold junction is usually placed in melting ice of a triple point cell. The Hot end is the probe and is put in the enclosure the temperature of which is to be measured. This temperature may be above or below the cold reference temperature.

The Emf produced across the Thermocouple is measured by potentiometer or high resistance voltmeter.

The Thermocouple itself consists of a one metal sandwiched between two others another.

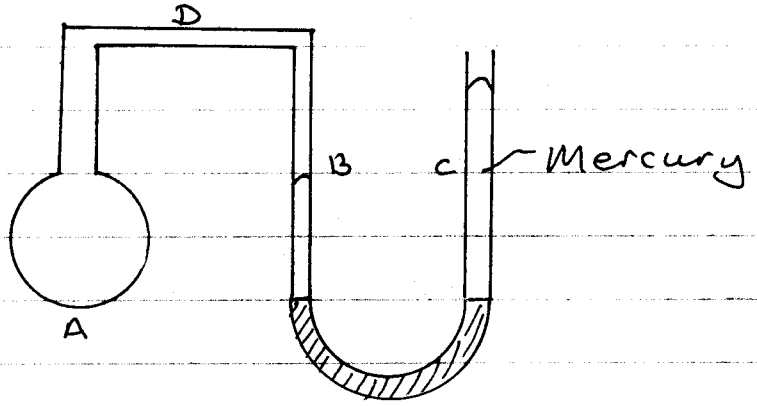
The Radiation Pyrometer.



This responds to radiation only. The current I is adjusted until the image of the filament cannot be distinguished.

Too low I filament appears black
Too high I filament lighter colour than surround.

Constant Vol Gas Thermometer



A simple constant volume thermometer above. The gas is air in simple models (helium/nitrogen for more accurate ones). As the temperature increases the gas expands pushing down mercury in B and up in C. By raising the Right hand column the volume is kept constant.

For accurate results we correct for:

(i) Gas in dead space D not at same temp as A
(ii) Thermal expansion of A
(iii) Capillary effects at mercury surfaces.

Thermal Conductivity

Heat Transfer.

Heat transfer is the passage of energy from a body at a higher temperature to one of a lower temperature.

Conduction of Heat. - Conductivity.

Consider a slab of Thickness δx
Cross Sectional Area A
Faces a small temperature difference $\delta\theta$
If δQ heat passes through slab

$$\text{Power} = \text{rate of doing work} = \frac{\delta Q}{dt}$$

$$\frac{dQ}{dt} = \text{Power} = -kA\frac{\delta\theta}{\delta x}$$

Negative sign indicates heat flow is towards the body at a lower temperature.

k the constant of proportionality is the rate of flow of heat through a material of unit area and temperature gradient.

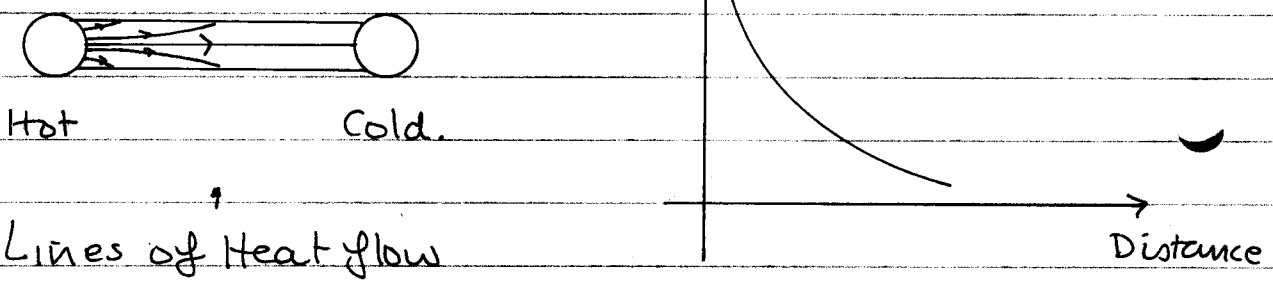
k = constant $[J\,s^{-1}m^{-1}C^{-1}]$ or $[Wm^{-1}C^{-1}]$

Temperature Gradients.

If heat is applied to a conductor for a considerable period of time an equilibrium state will be reached where the temperature at each point of the conductor becomes a constant.

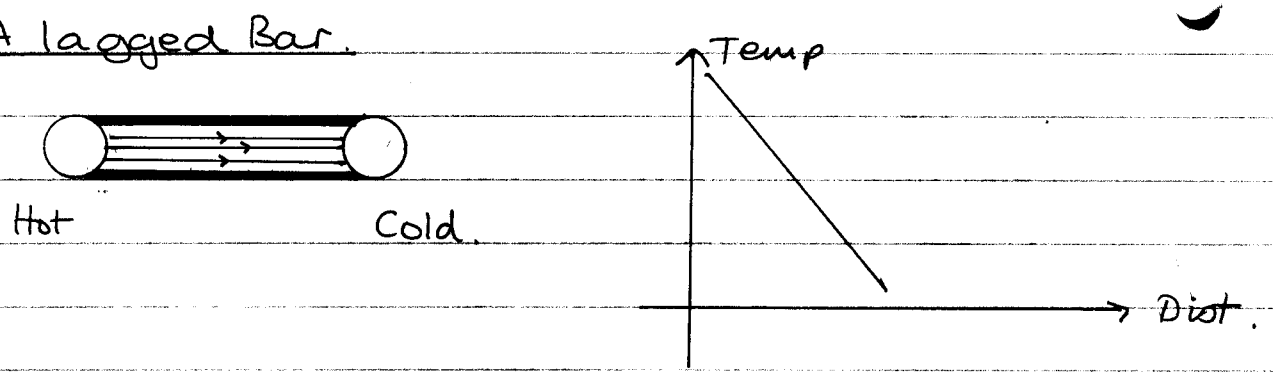
There are 2 cases to be considered:

1. An Unlagged Bar.



In an unlagged bar the quantity of heat passing through sucessive sections decreases due to the heat loss from the sides.
The temperature gradient is the tangent at that point i.e $\frac{d\theta}{dx}$.

2. A lagged Bar.



A lagged bar has its sides wrapped with a good insulator. Now the lines of heat flow are parallel and in the steady state the temperature falls at a constant rate. The tempure gradient is thus constant.

 M. Bennett.

Thermal Conductivity.

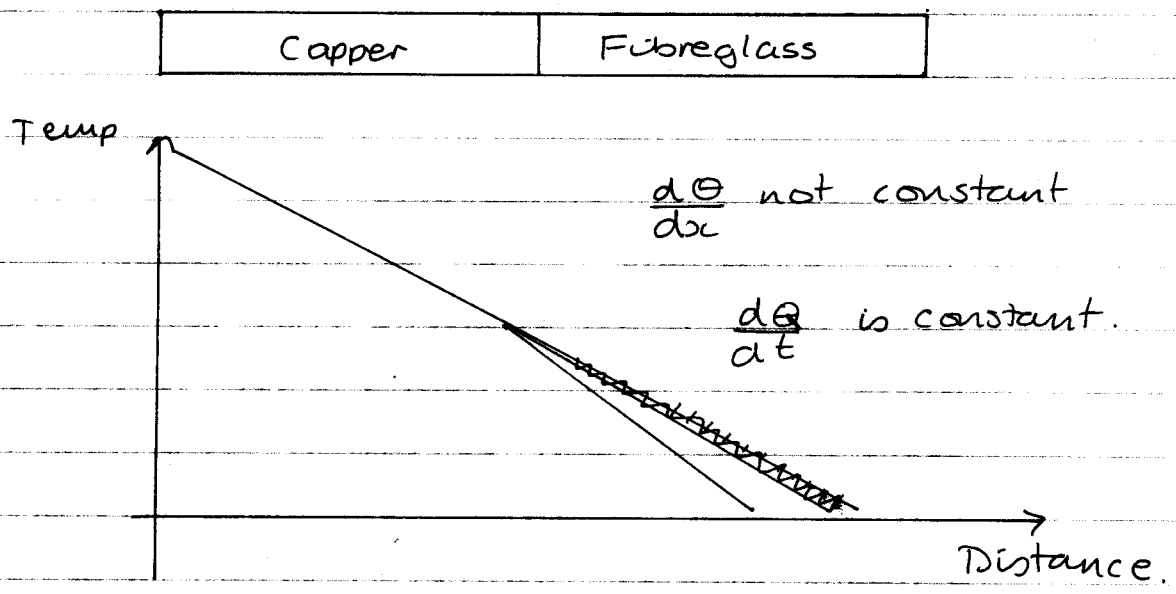
In a Lagged Bar we have Parallel heat flow.
We may determine a useful expression for this.

Consider a conductor of length x
Cross sectional Area A
Thermal conductivity K
Temperature across Hot, Cold ends θ_2 θ_1

$$\frac{Q}{t} = KA\left(\frac{\theta_2 - \theta_1}{x}\right)$$ Fouriers Law

For composite slab problem see 139
Materials + Mechanics Duncan.

Heat Loss in Compound Materials.



For a well laged constant area composite body $k\frac{d\theta}{dx}$ = constant.

$$K_1\left(\frac{d\theta}{dx}\right)_2 = k_2\left(\frac{d\theta}{dx}\right)_2$$

Mechanisms of Thermal Conduction

Conduction may occur due to Atoms, electrons or both.

In a Conductor atoms + electrons are involved.
In an Insulator only atoms conduct.

Conduction Via Atoms.

Atomic structures have a mean kinetic energy which is proportional to the temperature. At higher temperatures the atoms in the structures vibrate more vigourously about their mean positions.

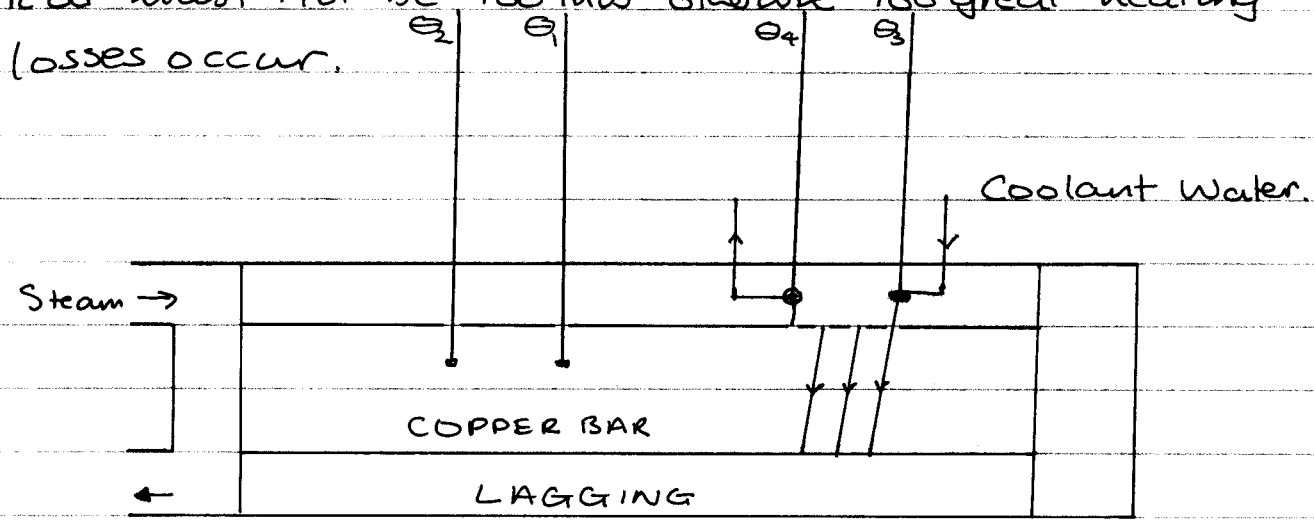
If heat is applied to a cold object then the increase of vibrational energy of the atoms is slowly transmitted throughot the object since the atoms are held via interatomic bonds. Since the atoms are quite heavy though this process is quite slow.

Conduction via electrons

The electrons share in any gain of energy due to the temperature rise in the material and are thus accelerated to a high degree due to their light weight. They can travel to all parts of the object quickly giving up their energy uppon collision with atoms. Hence the material will heat up quite rapidly.

Estimation of K for Good Conductors.

We use SEARLES method. On test is a bar of long length in comparison to diameter. This will give large x and $\theta_2 - \theta_1$ sufficiently big whilst keeping a satisfactory heat flow rate. Rod must not be too thin otherwise too great heating losses occur.



The Bar on test (Copper) is heavily lagged. It is steam heated at one end and water cooled at the other.

The inlet and oulet water coolent temperatures are noted so we can record the amount of heat leaving the bar.

We measure the temperature gradient by 2 thermometers θ_2, θ_1

If Q = quantity heat flowing down bar.

t = time, C = specific heat capacity water.

Assuming no power wasted:

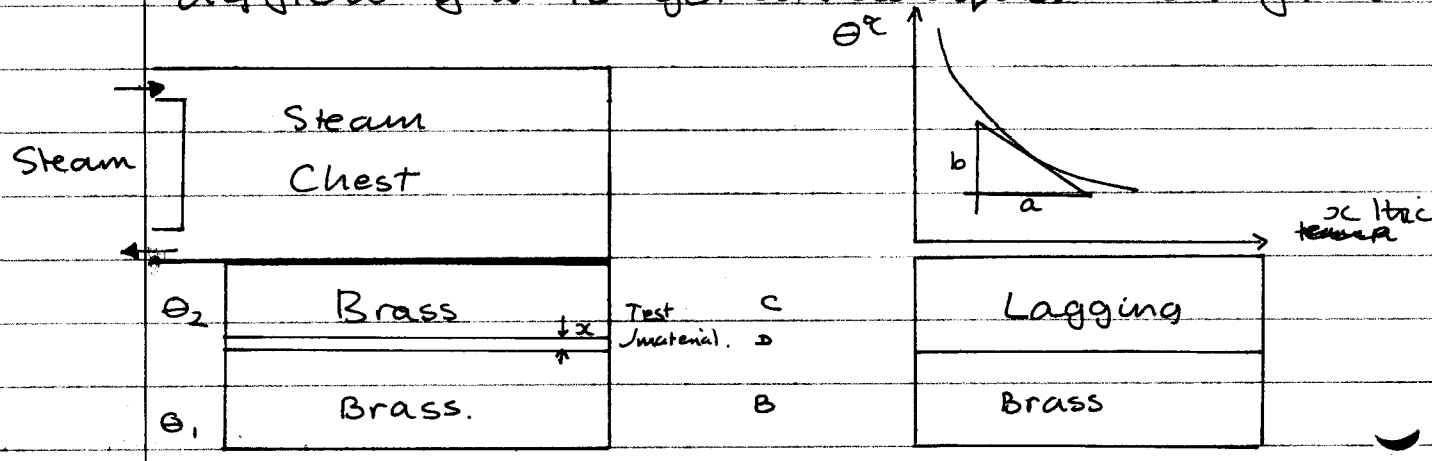
$$\frac{Q}{t} = kA\left(\frac{\theta_2 - \theta_1}{x}\right) = m\,c\,(\theta_4 - \theta_3)$$

where m = mass of water collected in t secs.

Hence K can be found.

K for Poor Conductors.

In this case even a thin same gives a measurable temperature gradient, the difficulty is to get an adequate heat flow



LEES DISC METHOD

The apparatus is circular in shape to reduce heat loss. The thin Disc under test rest in between 2 brass slabs, the top one being steam heated.

At first heat is passed through slabs until θ_2, θ_1 reach constant readings. At this point heat passing through D to B is the heat lost by B to surroundings. at θ_1, if this is Q then $Q = KA\left(\frac{\theta_2 - \theta_1}{x}\right)$

In the second part D is removed and B heated to a few degrees above θ_1 by C. B is then isolated and a cooling time curve plotted for B which is covered on top with asbestos.

Thus we can calculate $\frac{d\theta_1}{dt}$ for brass block

so

$$Q = mc\left(\frac{b}{a}\right) = -mc\frac{d\theta}{dt} = KA(\theta_2 - \theta_1)/x$$

where m = mass of Brass B, A = Area Disc

c = specific heat capacity of Brass, x = thick of Disc.

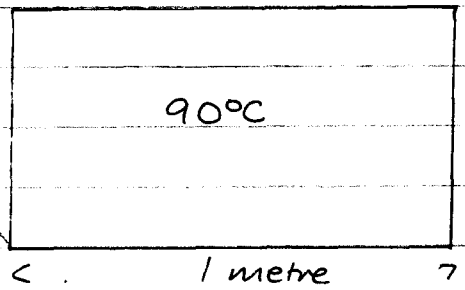
Thus k can be found.

P1. M.W.B.

11/10/79

P. Dunc P144 Nos 12-15.

(2)



Completely lagged (1.0cm)
Thermal conductivity
$= 6.4 \times 10^{-4}$ W/cm °C

Estimate the rate of flow of heat through the lagging if the external temp is 40°C.

Rate of flow heat $= P = kA(\theta_2 - \theta_1)/x$

$= 6.4 \times 10^{-2} \times 6(1 \times 1) \times (90-40)/0.01$

$= 1.92$ kW

Assuming lines of heat flow to be parallel.

Let New Thickness lagging be x cm.
Let Rate of flow of heat $= R$.

$\therefore R = 6.4 \times 10^{-2} \times 6(1 \times 1) \times (90-18) \times \frac{1}{0.01x}$

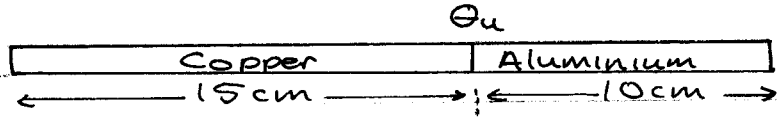
$R = \frac{2.76 \times 10^3}{x}$ Watts.

[I.E Power Dissip $\propto \frac{1}{x}$]

assuming this to be the case.

$R = \frac{2.76 \times 10^3}{x}$ watts.

13. The temperature gradient of a material is the rate of change of temperature with respect to distance moved through a conductor.



Free end of the copper is maintained at 100°C
Free end of the aluminium at 0°C.

Given $K_{copper} = 3.9\ W/cm°C$
$K_{Aluminium} = 2.1\ W/cm°C$.

Calc Temp gradient for each bar.

Because rate of heat flow is constant

$$\frac{d\theta}{dt} = -K A \frac{\Delta\theta}{x}$$

$$\therefore -3.9 \cdot A \cdot \frac{(100-\theta_u)}{15} = -2.1 \cdot A \frac{(\theta_u - 0)}{10}$$

$$\theta_u = \frac{390}{15} \Big/ \left(\frac{3.9}{15} + \frac{2.1}{10}\right) = 55.32°C$$

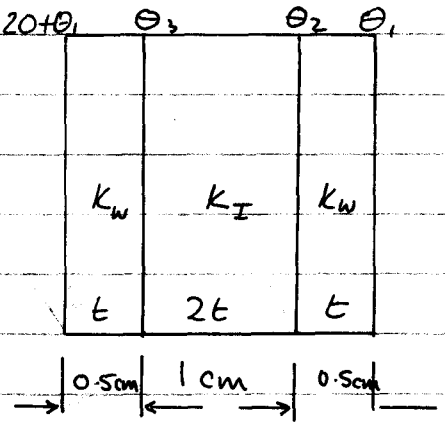
$$\therefore \text{Temp gradient copper} = \frac{100-55.32}{15} = 2.98°C\,cm^{-1}$$

$$\text{Temp gradient Aluminium} = \frac{55.32-0}{10} = 5.5°C\,cm^{-1}$$

Temperature gradients of Copper + Aluminium are $2.98°C\,cm^{-1}$ and $5.5°C\,cm^{-1}$ respectively

11/10/79.

Thermal Conductivity is the rate of flow of heat in that substance for a unit Area and a unit temperature gradient.



Heat internal temp = θ_1

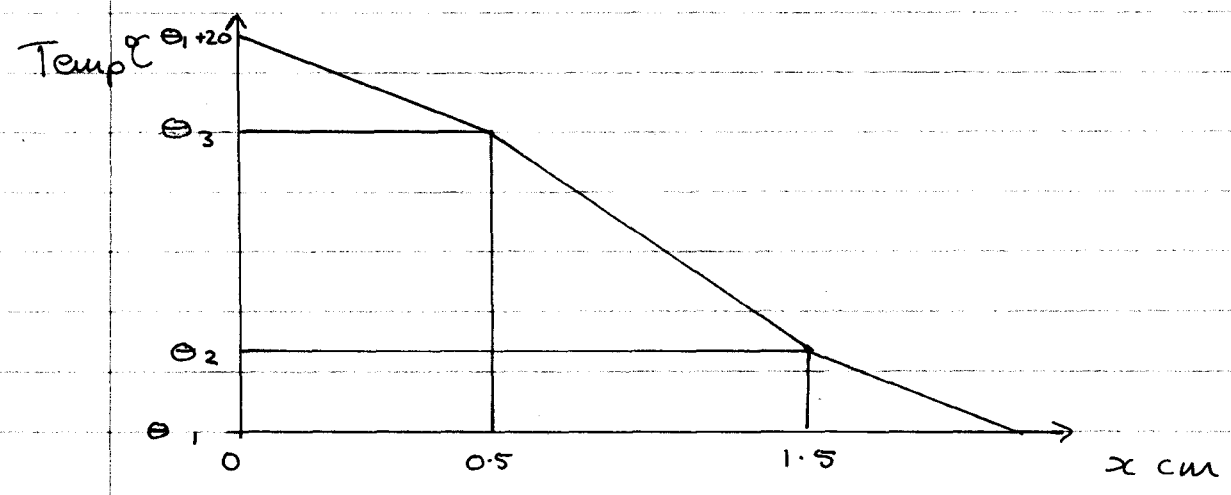
Temperature difference between external + inside surfaces is 20°C.

The rate of heat flow is the same through all parts of the insulating wall.

$$\frac{dQ}{dt} = P = -kA\frac{d\theta}{dx}$$

$$\frac{dQ}{dt} \text{ per unit area} = -k\frac{d\theta}{dx} = -k\frac{\Delta\theta}{x}$$

But $\frac{\Delta\theta}{x}$ is the same in both parts of the wood.



It works but not strictly correct.

Let $\theta_1 = 0°C$

Hence $k_w\left(\frac{20-\theta_3}{t}\right) = k_I\left(\frac{\theta_3-\theta_2}{2t}\right) = k_w\left(\frac{\theta_2-\theta_1}{t}\right) = \frac{k_w\theta_2}{t}$ ($\theta_1 = 0$)

$\Rightarrow 20-\theta_3 = \theta_2 \Rightarrow \theta_3 = 20-\theta_2$

$k_w\frac{\theta_2}{t} = k_I\left(\frac{\theta_3-\theta_2}{2t}\right) = k_I\left(\frac{20-2\theta_2}{2t}\right)$

$\Rightarrow 2\theta_2 k_w = k_I(20-2\theta_2)$

$2\theta_2(k_w+k_I) = 20k_I$

So $\theta_2 = \frac{10k_I}{k_w+k_I} = \frac{10(2.4\times10^{-4})}{(2.4\times10^{-4}+2.4\times10^{-3})}$

$\theta_2 = 0.909°C$

$P = \frac{dQ}{dt} \quad \therefore \frac{P}{A} = \frac{k_w\theta_2}{t} = \frac{10(2.4\times10^{-4})(24\times10^{-3})}{0.5(2.4\times10^{-4}+2.4\times10^{-3})}$

$= 4.36\times10^{-3}\ Wcm^{-2}$

Rate of heat flow per unit area $= 4.4\times10^{-3}\ Wcm^{-2}$ ✓

M.W.B

15

Since an equilibrium is in existance, the stirrer provides the water with the same amount of power as is being lost due to conduction of glass.

Thus $\frac{d\theta}{dt} = -kA\frac{\Delta\theta}{x}$

$\therefore \quad 75 = -0.84 \times 595 \times \left(\frac{1}{100}\right)^2 \times -\frac{(\theta - 15)}{0.2 \times 10^{-2}}$

$\Rightarrow \theta = 18.00 = 18.0\,^{\circ}C$

θ being Inside temp of flask.

Inside Temperature of flask = 18.0°C ✓

Thermal Expansion

1. Linear Expansion

The change of length which occurs with temperature is described with respect to linear expansivity α. If a solid length L increases by a length δL due to a temperature $\delta\theta$

$$\alpha = \frac{\delta L}{L} \times \frac{1}{\delta\theta}$$

Considering an original length L_0

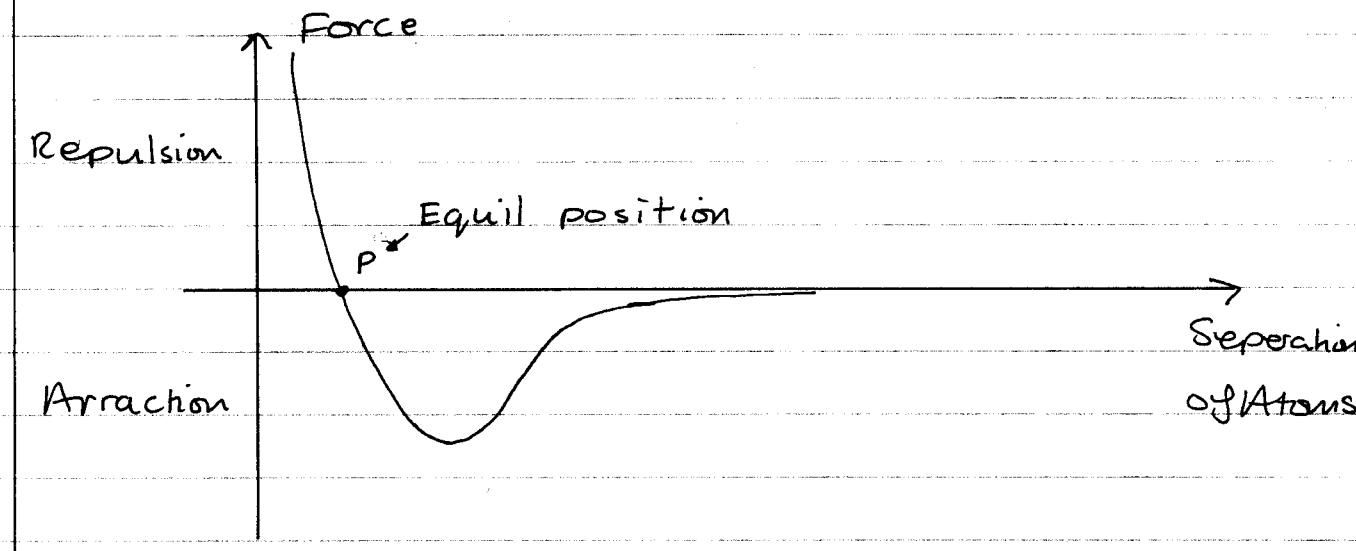
Length L_θ at θ °C

$$\alpha = \frac{L_\theta - L_0}{L_0 \theta}$$

$$L_\theta = L_0 (1 + \alpha\theta)$$

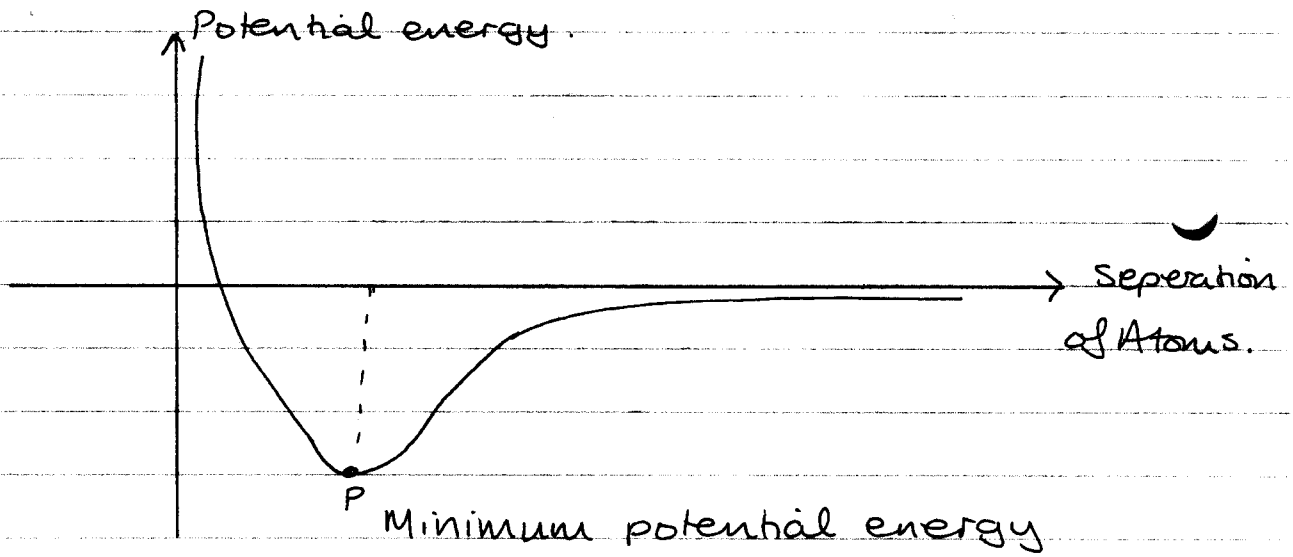
Thermal expansion on an atomic scale.

We can explain this on an atomic scale by thinking of the sum of the repulsive and attractive forces between atoms.

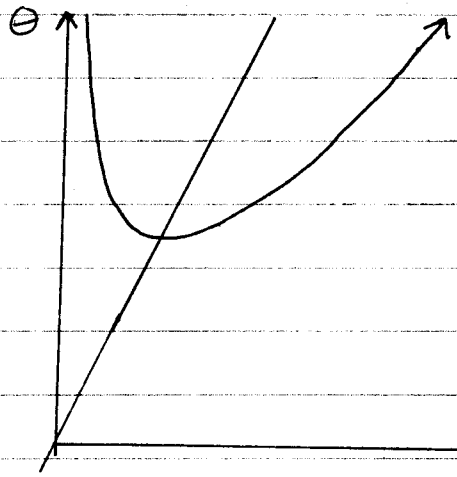


A rise in temperature increases the amplitude of vibration of the atoms in the crystal lattice about their equilibrium position and the average separation increases. At low temperatures the oscillation is small and symmetrical.

Potential Energy of the Atoms.



Minimum potential energy (the P on the previous diagram).



Around the minimum PE mark a temperature increase gives a nearly linear increase in MEAN particle separation.

Mean sep ∝ Length Temp ∝ Potential energy.

Connection between Youngs modulus and coef of Linear Expansion

A bar of original length L
Area A
coef expansion α
has been extended by e

α is defined as the fractional increase in length per degree rise in temperature.

$$\alpha = \frac{e}{L\theta}$$ where θ is the temp rise.

so $e = \alpha L \theta$

But we know $E = \frac{FL}{Ae}$

E = Youngs modulus F = Force.

$$\therefore e = \alpha L \theta = \frac{FL}{AE}$$

$$\therefore F = EA\alpha\theta$$

i.e Force F produced same expansion as temp rise θ $EA\alpha$ being a constant for a given material.

e.g. What force is needed to stop a steel rail expanding when θ change is 40°C

P.T.O.

$F = E A \alpha \theta$

$F = ?$ $\quad E = 2 \times 10^{11}\ Pa$ $\quad \alpha = 1.2 \times 10^{-5} K^{-1}$

$A = 100 cm^2$ $\quad \theta = 40^{\circ}C$

$F = E A \alpha \theta = 9.6 \times 10^{5}$ Newtons.

The kinetic Theory.

The kinetic Theory of an Ideal Gas.

The kinetic theory attempts to relate the macroscopic behaviour of an ideal gas with the microscopic properties e g speed, mass of its molecules.

Given certain assumption we can create a model and obtain results based on that model

We assume:

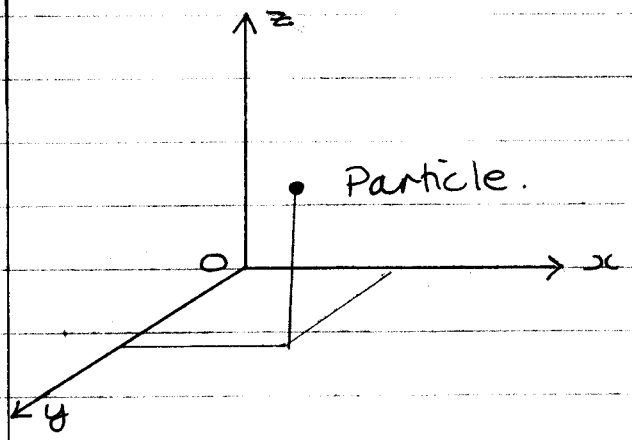
a The range of intermolecular forces is small compared with the average distance between the molecules. From this it follows:

(i) The intermolecular forces are negligable except in a collision

(ii) The volume of the molecules themselves is negligable compared with the volume of the gas.

(iii) The time occupied by a collision is negligable compared with the time spent between collisions.

(iv) between collisions molecule moves with uniform velocity.



We consider a cubical box side l containing N molecules of mass m each. It moves with resultant velocity c which may be resolved into u, v, w in directions x, y, z.

Consider motion along Ox. When it strikes wall it rebounds elastically.

change momentum $= mu - (-mu) = 2mu$.

Speed of molecule $= \frac{x}{t} \; u$.

$$\text{accl} = \text{Rate change momentum} = \frac{2mu}{2l/u} = \frac{mu^2}{l}$$

$\therefore$ Total force in x direction

$$= \frac{m}{l}\left(u_1^2 + u_2^2 + u_3^2 \ldots u_N^2\right)$$

but since $\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$ u, v, w all being equal due to random motion

$$\therefore \quad \frac{\overline{c^2}}{3} = u^2$$

$$\therefore \text{ Total force} = \frac{m}{3l}\left(c_1^2 + c_2^2 + c_3^2 \ldots c_N^2\right)$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{m}{3l^3}\left(c_1^2 + c_2^2 + c_3^2 \ldots c_N^2\right)$$

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$$\overline{c^2} = \frac{(c_1^2 + c_2^2 + c_3^2 + c_4^2 \ldots c_N^2)}{N}$$

$$\therefore \text{Pressure} = p = \frac{m}{3l^3}(c_1^2 + c_2^2 + c_3^2 + c_4^2 \ldots c_N^2)$$

$$p = \frac{Nm\overline{c^2}}{3l^3}$$

$$P = \text{Pressure} = \frac{1}{3}\frac{Nm\overline{c^2}}{L^3} = \frac{1}{3}\frac{Nm\overline{c^2}}{V}$$

N = total number of molecules
m = mass of a molecule
$\overline{c^2}$ = mean square speed
V = total volume of enclosure.

Alternatively $p = \frac{1}{3}\rho\overline{c^2}$

where ρ = density $= \frac{Nm}{V}$

Derive $PV = nRT$

since

$$PV = \frac{1}{3}Nm\overline{c^2}$$

$$PV = \left(\frac{2}{3}N\right)\left(\frac{1}{2}m\overline{c^2}\right)$$

$PV \propto$ mean KE of molecules.

i.e Temperature varies as KE

$$T \propto \frac{1}{2}m\overline{c^2}$$

$PV \propto T$

$PV = nRT$

$PV = nRT$

P = Pressure

V = Volume.

n = Number of moles

R = gas constant

T = temperature in Kelvin.

 M. W. Bennett.

P373 Q1, 5, 6, 8

Q1

1 mole oxygen occupies $22.4 dm^3$ at STP

Calc R = the gas constant.

$$R = \frac{PV}{nT} = \frac{1 \times 10^5 \times 22.4 \times 10^{-3}}{273} \quad 8.21$$

Gas Constant = 8.21

⑤ Density of nitrogen $= 1.25 kg\, m^{-3}$

Calc root mean square speed of Nitrogen molecules at 227°C.

$$P = \frac{1}{3} \rho \overline{C^2} \Rightarrow \overline{C_s^2} = \frac{3P}{\rho} = 2.4 \times 10^5$$

But $\frac{1}{2} m \overline{C^2} \propto T$ $\therefore$ $\overline{C^2} \propto T$

$$\therefore \frac{\overline{C_s^2}}{\overline{C_A^2}} = \frac{273}{227+273}$$

$\overline{C_A^2} = 4.39 \times 10^5$ RMS $= \sqrt{4.39 \times 10^5} = 6.63 \times 10^2 ms^{-1}$

RMS speed of Nitrogen $= 6.63 \times 10^2 ms^{-1}$

6 A closed vessel contains hydrogen which exerts a pressure of 20.0 mmHg at a temp of 50.0K. At what temp will it exert 180 mm Hg.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \Rightarrow T_2 = \frac{180}{20} \times 50 = 450K$$

Temperature for 180 mm Hg pressure = 450 K.

RMS velocity of the hydrogen molecules was 800 ms^{-1} what is new velocity?

$$\frac{1}{2} m \overline{C^2} \propto T$$

$$\therefore \frac{\overline{C_A^2}}{\overline{C_B^2}} = \frac{T_A}{T_B}$$

$$V = \sqrt{\left[800^2 \times \frac{450}{50}\right]} = 2400 ms^{-1}$$

New velocity = 2400 ms^{-1}

8. Vessel of volume 50 cm^3 at Pressure 1.0 Pa at 27°C

(a) Number of molecules in vessel

$PV = nRT$ ∴ No molecules

$$= n \times 6 \times 10^{23} = 6 \times 10^{23} \times \frac{1 \times 50/(100)^3}{8.3 \times 300} = 1.2 \times 10^{16}$$

No of molecules = 1.2×10^{16}

M. W. Bennett

(b) Calculate their mean separation.

Assuming even spacing each molecule is in the centre of a cube of volume $\frac{50}{1.2 \times 10^{16}}$ cm^3

i.e $4.16 \times 10^{-15} cm^3$

Distance between molecules is numerically equivalent to the length of a cube side

i.e $\sqrt[3]{4.16 \times 10^{-15}} = 1.6 \times 10^{-5} cm$

$= 1.6 \times 10^{-7} m$

Distance between molecules $= 1.6 \times 10^{-7} m$

(c) Calc root mean square speed.

$PV = \frac{1}{3} N m \overline{c^2}$

$\therefore \sqrt{\overline{c^2}} = \sqrt{(3PV/Nm)}$

$= \sqrt{3.1 \times \frac{50}{100^3} \Big/ \frac{2 \times 10^{-3}}{6 \times 10^{23}} \times 1.2 \times 10^{16}}$

$= 1.94 \times 10^3 \, ms^{-1}$

Root mean square speed $= 1.94 \times 10^3 \, ms^{-1}$

Deductions from the kinetic Theory.

a Avogadro's Law

Equal volumes of ideal gases at the same temperature and pressure contain the same number of molecules.

(b) Daltons Law of Partial Pressures

The total pressure exerted by a mixture of gases in a container is equal to the sum of the partial pressures.

$P_1 V = \frac{1}{3} N_1 m_1 \overline{c_1^2}$ $\qquad P_2 V = \frac{1}{3} N_2 m_2 \overline{c_2^2}$

but since $T_1 = T_2$ $\quad \frac{1}{2} m_1 \overline{c_1^2} = \frac{1}{2} m_2 \overline{c_2^2}$

so $PV = \frac{1}{3} N m \overline{c^2}$, $m \overline{c^2} = m_1 \overline{c_1^2} = m_2 \overline{c_2^2}$

$\therefore N = N_1 + N_2$

$PV = \frac{1}{3} (N_1 + N_2) m \overline{c^2} = P_1 V + P_2 V$

$\therefore \underline{P = P_1 + P_2}$

(c) Graham's law of diffusion

We assume the rate of diffusion is directly proportional to the size of the hole, and the rate at which molecule hits the side with the hole.

$$\frac{\text{Rate diffusion 1}}{\text{Rate diffusion 2}} = \sqrt{\frac{\overline{C_1^2}}{\overline{C_2^2}}}$$

But, from $P = \frac{1}{3} \rho \overline{c^2}$

$$P_1 \overline{C_1^2} = P_2 \overline{C_2^2}$$

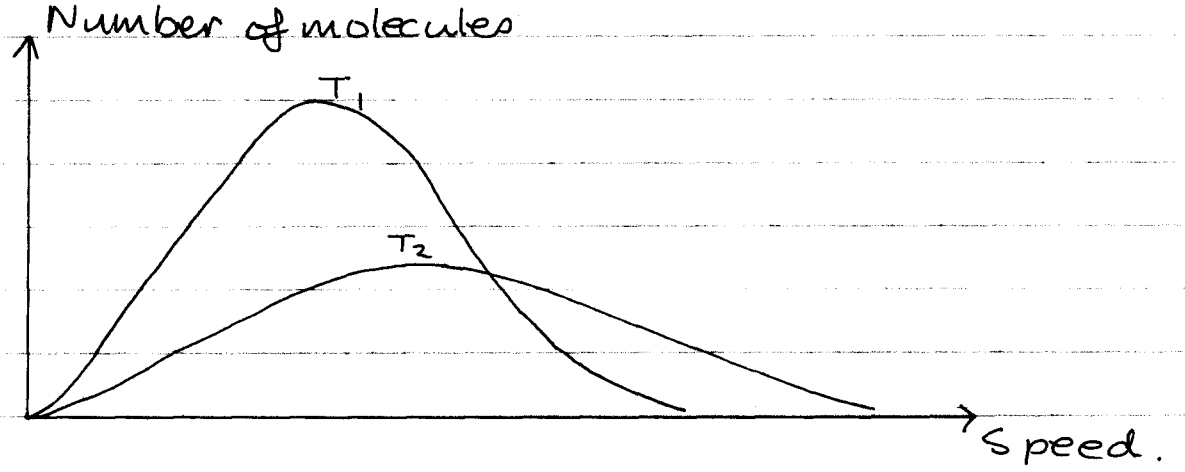
$$\frac{\overline{C_1^2}}{\overline{C_2^2}} = \frac{P_2}{P_1}$$

$$\text{Rate of diffusion}\left(\frac{1}{2}\right) = \sqrt{\frac{P_2}{P_1}}$$

Rate of diffusion is inversely proportional to the square root of its density.

Boiling Evaporation + Kinetic Theory.

Molecules within the body of a liquid are surrounded by others which move at random with a large distribution of speeds.



Molecules near the surface do no have as many molecules surrounding them on top. Thus there is a resultant force down into the liquid.

A molecule near the surface experiences a net resultant force towards the body of the liquid. This explains the phenomena of SURFACE TENSION.

A molecule on the surface has done work against the intermollecular forces and so has more potential energy. This energy comes from the distribution of energies of the molecules.

Creating more surface area results in a cooling of the liquid. If a high energy molecule is moving towards the surface it has work done on it by the intermollecular

forces and it does work against them by surface tension. In doing so it escapes into the space around it.

On increasing the temperature, the number of high energy molecules increases and hence the mean molecular separation. These 2 factors substantially increase the rate of evaporation (non linear).

The escaped molecules may leave for good particularly if a wind is blowing but in a confined space as the number of molecules in the same space increases the pressure exerted by the gas molecules (vapour pressure) increases so the rate of collision of gas molecules increases.

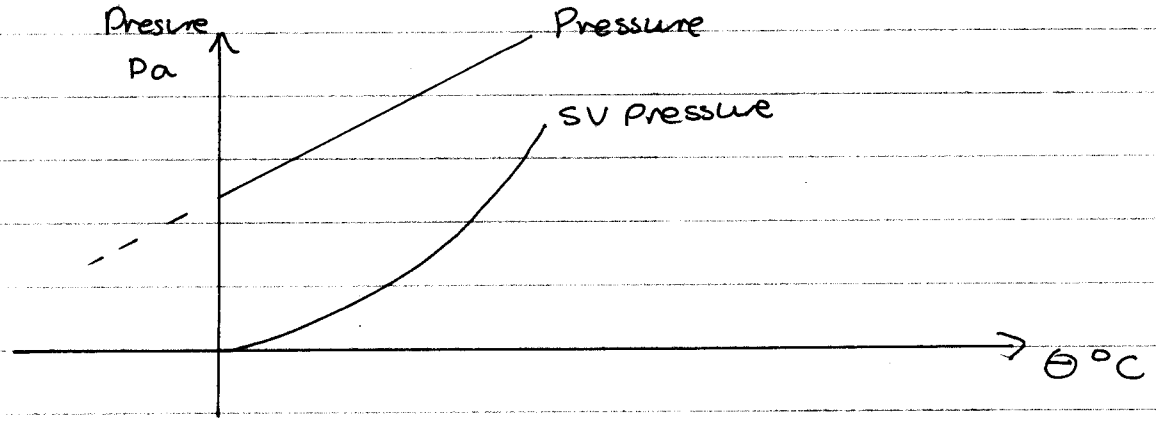
An equilibrium is thus reached and at this point the vapour exerts the saturated vapour pressure.

Mixture of Gases + Sat Vapours.

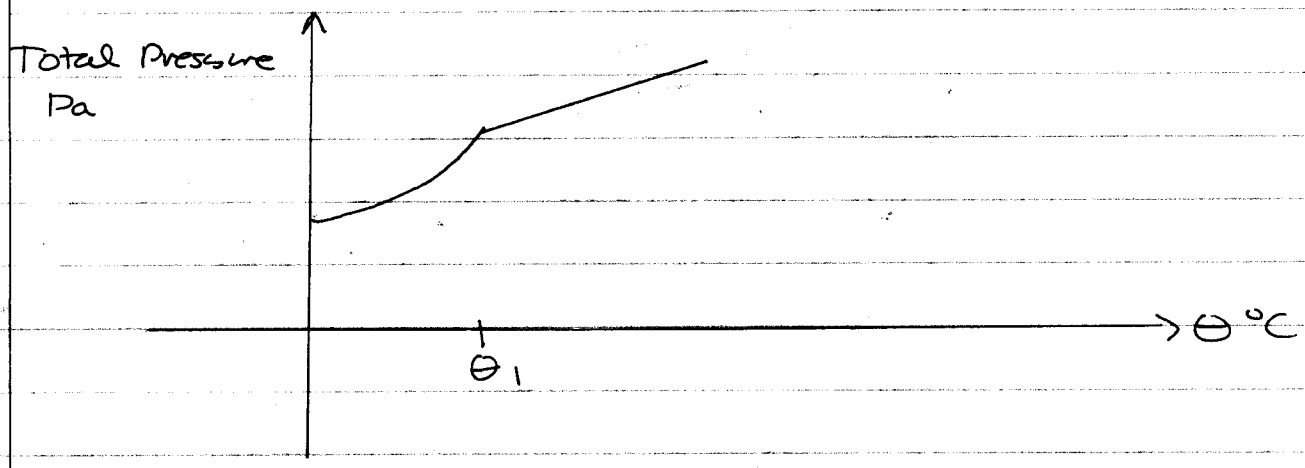
If the space above a liquid contains a gas as well as a saturated vapour then the pressure is found to be the sum of the partial pressures due to the gas (TnR/V) and by the SVP.

Gases + Vapours.

For a closed container with a gas vapour and some liquid.

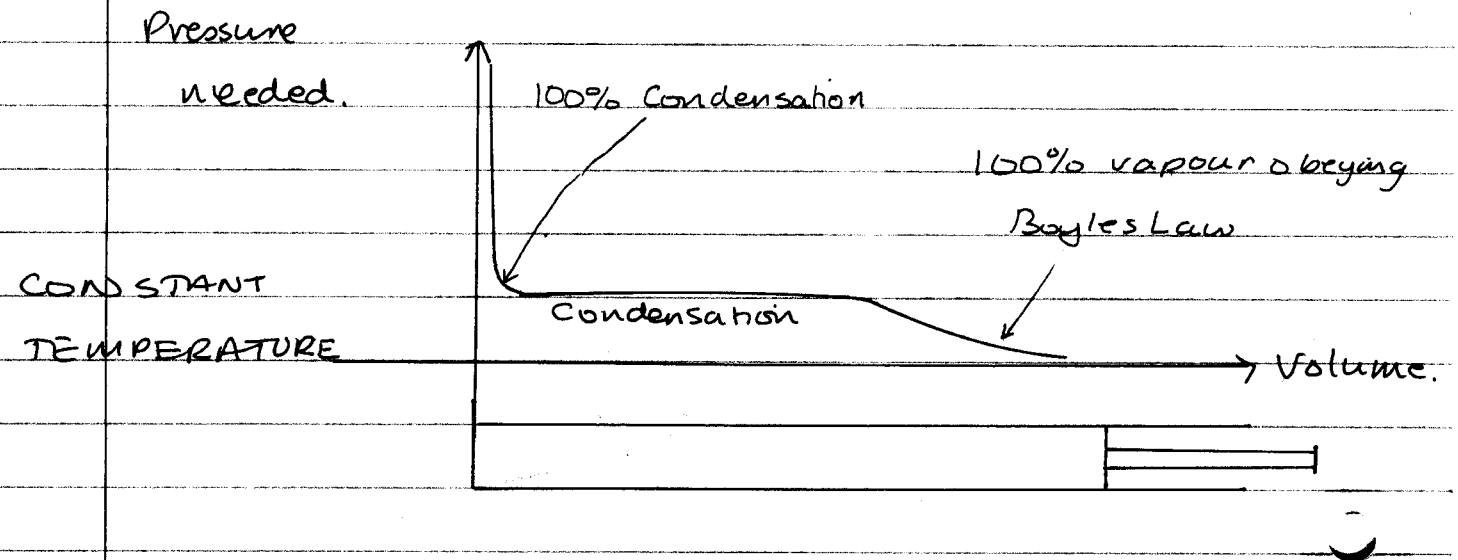


Total Pressure Curve.



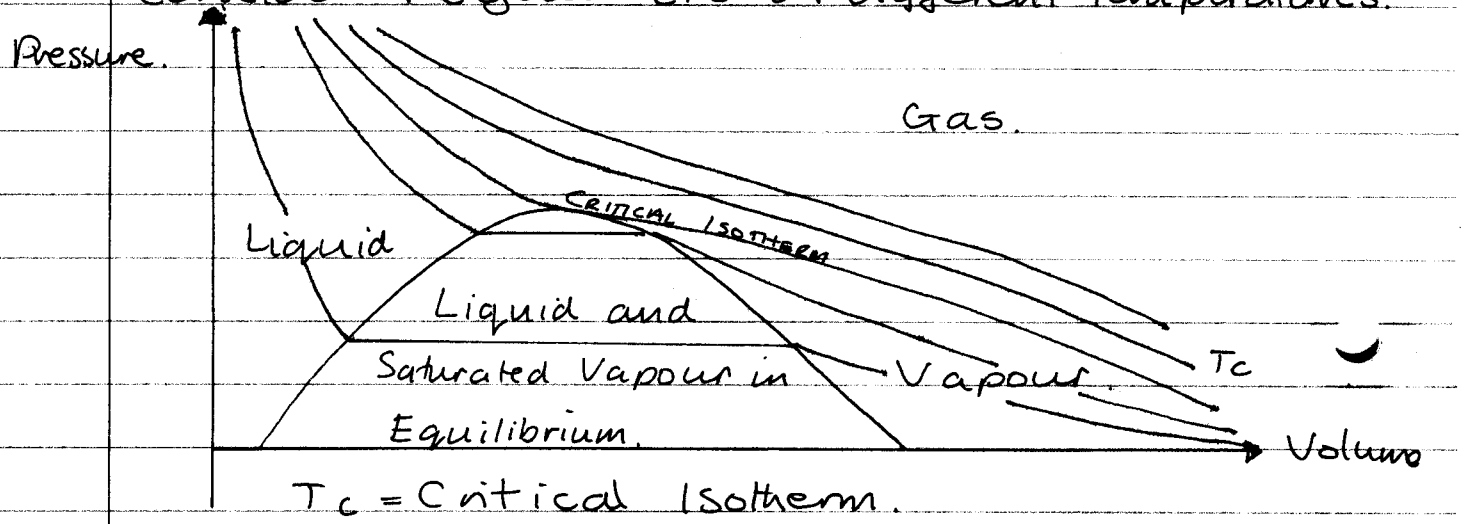
Providing that some liquid remains the total pressure is given by the curve A. If at temp θ, all the liquid is vapourised then above this temperature the vapour is no longer saturated and obeys the gas laws. Each partial pressure P_{gas}, $P_{unsat\ vapour}$ obeys the gas laws, and so total pressure obeys gas laws.

Variation between pressure and volume for saturated and unsaturated vapours.



We see variation of pressure as volume gradually increases.

A family of curves is obtained when we consider the gases etc at different temperatures.



T_c = Critical Isotherm.

A material in the gaseous phase above a certain critical temperature cannot be liquified by the application of any high pressure alone. Such a material is called a gas. Below the critical temperature the material in the gaseous phase is a vapour.

e.g T_c oxygen $-119°C$

carbon dioxide $31°C$

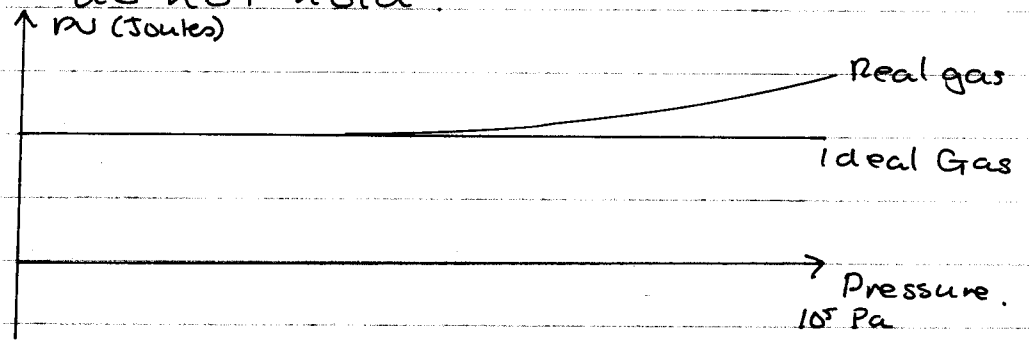
Water $374°C$

Deviations from the Gas Laws

For all Real gases:

(a) $\frac{PV}{\mu T}$ does vary with temperature and pressure.

(b) The assumptions of the theory that led to $p = \frac{1}{3}\rho \bar{c}^2$ do not hold.



For real gases PV increases with P provided the pressure is large enough.

2 possible Breakdowns of the Kinetic Theory.

1. The repulsive intermolecular forces are operative at relitavely large distances e.g. the volume of the actual molecules is comparible with the volume of the gas.

2. The attractive intermolecular forces are operative at relitavely large distances. This can cause the formation of molecular complexes and the result is a reduction in the pressure exerted by the gas.

Van de Waals Equation.

$$p_i\left(\frac{V_i}{\mu}\right) = RT$$

Equation of state for an amount μ of an ideal gas.

1. Effect of Repulsive intermolecular forces.

Consider 1 mole of real gas occupying volume V_m. The actual volume of gas molecules reduces the volume by b (the molar covolume)

$V_m - b = \frac{V_i}{\mu}$ is the molar volume which can be altered by changes of temperature and pressure and which corresponds to the molar volume of an ideal gas.

2. Effect of Attractive forces

Attractive forces between non ideal gas molecules may cause 2 or more molecules to become bound together in dimers.

Thus the total number of particles is reduced so is the number of moles.

$\mu_{real} < \mu_{ideal}$.

p_i = pressure that would have been measured if it was an ideal gas.

p = pressure actually measured.

p_a = correction to be added to make p to p_i

$$p_i = p + p_a$$

We say $p_a = \frac{a}{V_m^2}$ (QUOTE ONLY) $\quad p_i = p + \frac{a}{V_m^2}$

Deviations from Gas Laws.

3. Effect of Container walls.

A molecule on approaching a container wall experiences an attractive force towards it but also when it rebounds and thus the forces cancel and the wall has no net of effect on the modulus of momentum of the particle.

Summary.

$$p_i \left(\frac{V_i}{\mu} \right) = RT$$

$$\left(P + \frac{a}{V_m^2} \right) (V_m - b) = RT$$

Substance.	$a / Nm^4 mol^{-2}$	$b / m^3 mol^{-1}$
Hydrogen	2.5×10^{-2}	2.7×10^{-5}
Nitrogen	1.4×10^{-1}	3.9×10^{-5}
Carbon Diode	3.6×10^{-1}	4.3×10^{-5}

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P373 Q3,4 Definitions + Calculations.

(1) Distinguish Between Vapour Pressure + SVP.

If a small quantity of a volatile liquid is placed in a vacuum is evaporates and exerts a vapour pressure. The pressure exerted is dependent upon the amount of liquid added. When SVP occurs this pressure is at a maximum and the pressure exerted by the vapour is in equilibrium with the liquid.

Calculation.

Closed vessel fixed volume contains air/water

$P = 737.5 \text{ mm Hg}$ @ $293K$

$P = 1144 \text{ mm Hg}$ @ $348K$

SVP at 20°C is 17.5 mm Hg. Find it at 75°C.

In the Air vapour mix Pressure exerted by air is $P - SVP_T$

$$\therefore \quad \frac{P - SVP_{T_1}}{T_1} = \frac{P - SVP_{T_2}}{T_2}$$

$$\Rightarrow SVP_{T_2} = -\left[\frac{737.5 - 17.5}{293} \times 348\right] + 1144 = 288.8 \text{ mm}$$

Saturation Vapour Pressure at 75°C is 288.8 mm

(4) A saturated vapour is one in which for a given Pressure + Volume a vacuum contains the maximum amount of vapour possible. At this point if more liquid is added it will not vaporise. In a saturated vapour a dynamic equilibrium occurs between liquid and vapour molecules.

Calculation.

A flask contains a mixture air / unsat water vapour.

$T = 323K \quad P = 8 \times 10^2$ mm Hg.

Mixture cooled when $T = 293K$ SVP $= 18 \times 10^0$ mm Hg

$T = 280K$ SVP $= 7.0 \times 10^0$ mm Hg

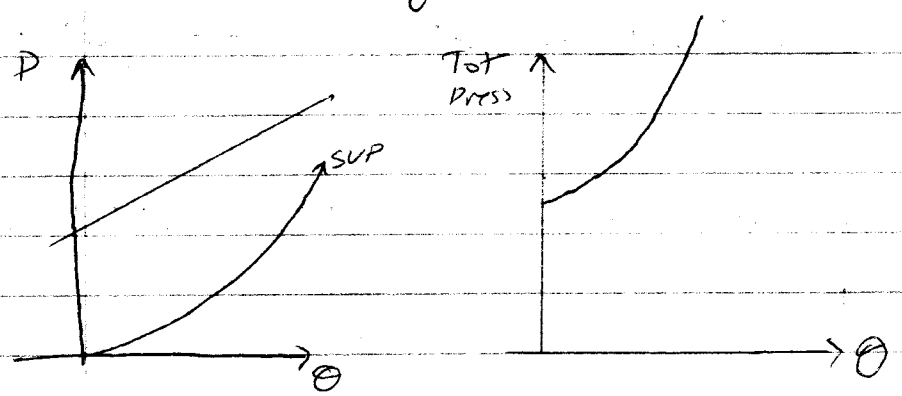
Calc Pressure of mix at 5°C assuming vapour obeys gas laws upto point of Saturation.

$$P_{293K} = 800 \times \frac{293}{323} = 725.7 \text{ mm Hg.}$$

Let x be pressure at 5°C.

$$\frac{725.7 - 18}{293} = \frac{x - 7}{280} \Rightarrow x = 633 \text{ mm Hg.}$$

Pressure of mixture at 5°C is 630 mm Hg (2SF).



Thermodynamics, Work + Energy of Gases.

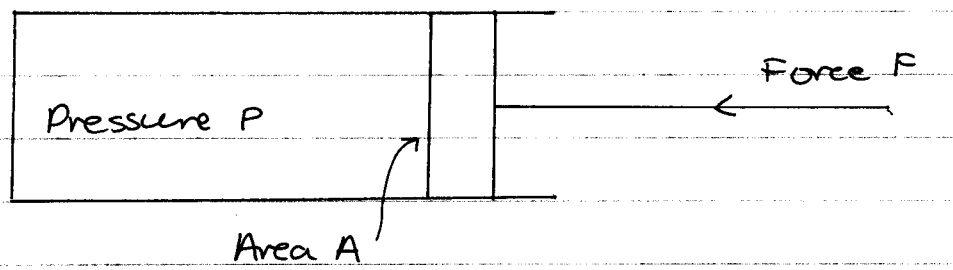
Internal Energy
The total energy of all the molecules of the gas.

Translational Energy
The energy considered in the kinetic theory.

Rotational and Vibrational energy of polyatomic molecules

Intermolecular potential energy.
This only occurs if work has to be done to separate molecules. This is zero in an ideal gas.

Work Done by or on a Gas.



Consider a mass of gas held in a cylinder by the piston. The gas has a volume V and exerts a pressure P.

Force exerted on piston = $P \times A$

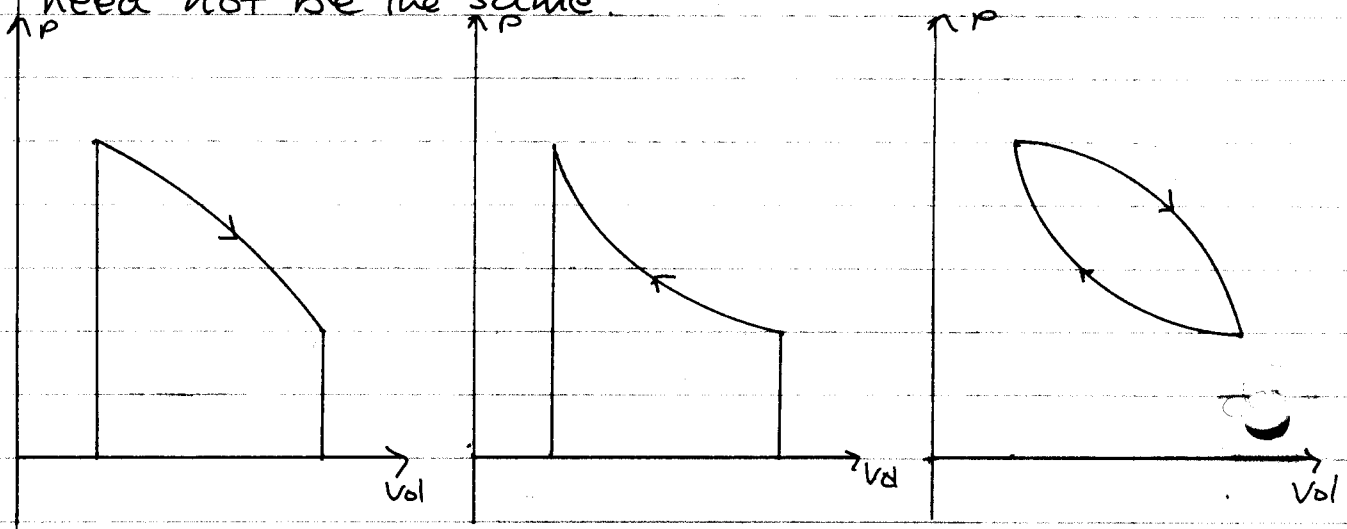
Work for movement δx is $\delta W = F \delta x$

So if the volume of the piston enlarges from V_1 to V_2 then work is done by the gas thus reducing its own internal energy.

$$\text{Work} = W = \int dw = \int F \delta x = \int PA \, dx$$

$$W = \int_{V_1}^{V_2} P.(A\,dx) = \int_{V_1}^{V_2} P\,dV$$

The Work done on or by a gas in not a unique curve. If the gas is recycled a closed loop is formed but the expansion an compression graphs need not be the same.



Work done by Gas on surroundings

Work done on gas by surroundings

Nett work done by gas.

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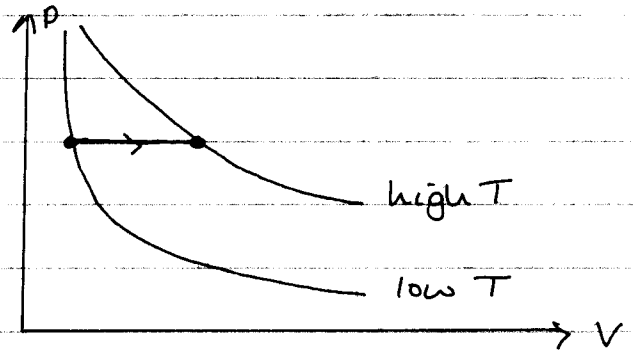
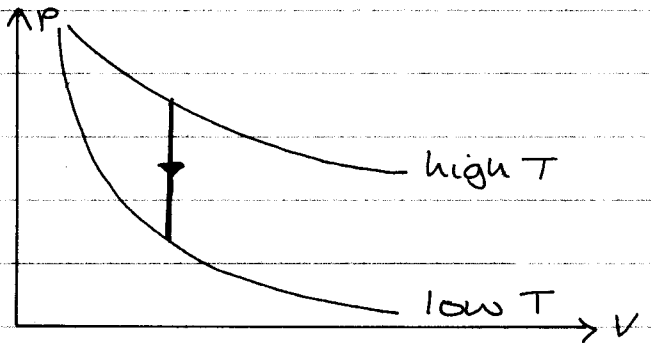
Reversible Gas changes.

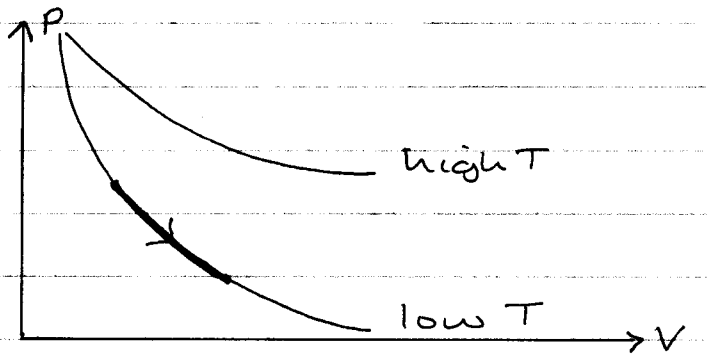
1. Isobaric process
pressure constant.



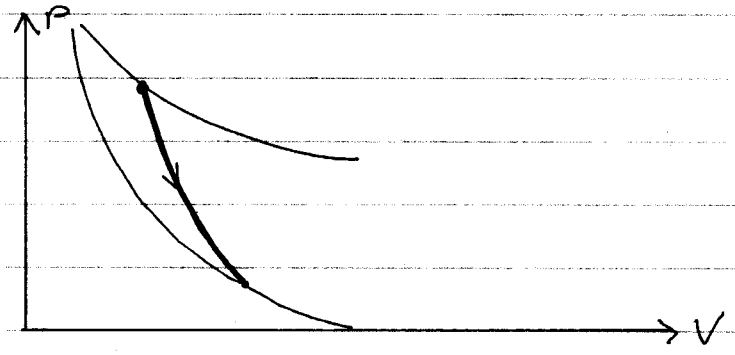
2. Isovolumetric Process
Volume constant



3. Isothermal Process
Temp constant
PV constant.



4. Adiabatic Process
$PV^{\gamma} = \text{constant}$.



Thermo Dynamics.

When heat is supplied to a body the increase in energy can appear in 2 forms internal and external energy.

$$\delta Q = \delta U + \delta W$$

Heat supplied = Internal energy + external energy (work)

↑ Poss temp in or change state.

↑ Increase in dimension of body

Thermo Dynamics for an Ideal Gas.

For an ideal gas $PV = RT$

Also for a constant volume no work is done i.e $\delta W = 0$

Thus for δQ heat input

δu in temal energy

$$\delta Q = \delta u.$$

Let C be the molar heat capacity. This is the energy in joules required to raise 1 mole of gas through 1°C. (C_v for constant volume)

$$\delta Q = C_v \, dt$$

The same gas recieves heat but its volume is allowed to increase to keep presure constant.

$$\delta Q' = C_p \delta t$$

If the gas has the same rise in temperature more heat will be required this time since external work δW is also done

$$\delta Q' = \delta u + \delta w$$

$$\therefore \quad C_p \delta t = \delta Q' = du + \delta w$$

$$C_p \delta t = C_v \delta t + P \delta V$$

i.e Work Done $= P \delta V = \delta W = (C_p - C_v) \delta t$

But $PV = RT$ so $P \delta V = R \delta t$

$$R \delta t = (C_p - C_v) \delta t$$

$$\underline{R = C_p - C_v}$$

This is known as Mayers Relation for an ideal gas.

Evidence for this supposition can be found by collating the relevant experimental data.

For an ideal gas

$$C_p = 20.78$$

$$C_v = 12.47$$

$$C_p - C_v = 8.31 = R.$$

C_p and C_v from the kinetic Theory.

From the kinetic Theory $PV = \frac{1}{3}Nm\overline{c^2}$

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{c^2}\right) = \frac{2}{3}N(\text{mean kE of molecule})$$

There are N_A (Avogadros number) molecules in a mole

$$\therefore PV = \frac{2}{3}n\left(\frac{1}{2}N_A m\overline{c^2}\right) \quad \text{since } N = nN_A$$

$$PV = \frac{2}{3}n(\text{KE of 1 mole}) = nRT = \frac{2}{3}n(E_k)$$

$$E_k = \frac{3}{2}RT$$

$$\text{KE of 1 mole} = \frac{3}{2}RT$$

At constant volume $\delta Q = \delta U$

for 1 mole $\delta U = \delta E_k = \frac{3}{2}R\delta t$

$$C_v = \frac{\delta U}{\delta t} = \frac{\text{Change of energy}}{\text{Change of temp.}}$$

$$C_v = \frac{dU}{dT} = \frac{3}{2}R$$

$$C_v = \frac{3}{2}R = 12.47\,\text{J mol}^{-1}\text{k}^{-1}$$

For monatomic gases only.

Monatomic Gases

I.e one atom per molecule

$$ke_{mean}\ (\text{of 1 molecule}) = \frac{1}{2} m\overline{c^2} = \frac{1}{2} m(\overline{u^2}+\overline{v^2}+\overline{w^2})$$

There are 3 mutually perpendicular directions in space in which the kinetic energy of translation may exist. (Only form energy)

As each direction is equally likely we say that the kinetic energy exists in 3 degrees of freedom.

Each Direction takes an equal proportion of the energy.

Diatomic Gases

For a diatomic gas there are 5 degrees of freedom 3 translational and 2 translational No ~~rotation~~ rotation occurs about the dumb bell axis and no additional vibration is excited near room temperature for strongly bound molecules such as N_2, O_2

Average energy = $\frac{5}{2}kT$ per molecule.

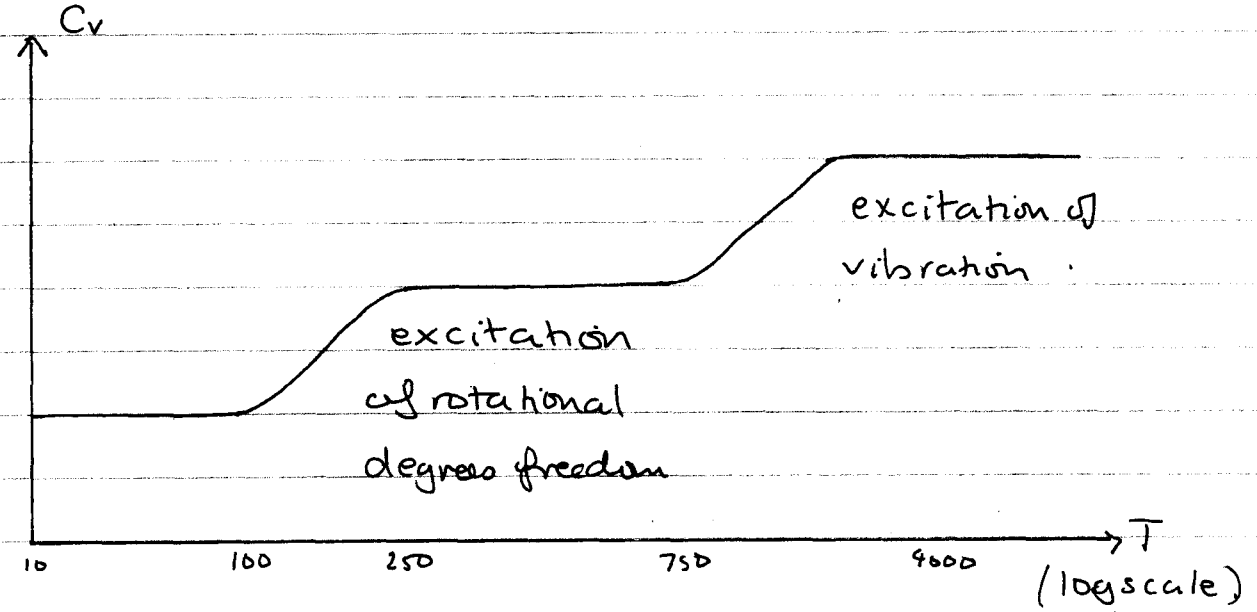
$$\therefore C_V = \frac{5R}{2} = 20.8 \text{ J mol}^{-1}\text{K}^{-1}$$

in general $C_V = \frac{nR}{2}$ where n = number degrees freedom.

However C_V varies with T. At low temperatures no rotation modes are ~~x~~ excited.

Medium temp excitation of rotational degrees of freedom.

High temp - excitation of vibration.



Summary.

1. In a monotomic gas $C_v = \frac{3R}{2}$ (3dof)

$$C_p = R + C_v \Rightarrow C_p = \frac{5R}{2}.$$

2. For a diatomic gas.

$$C_v = \frac{5R}{2} \quad (5\text{dof})$$

$$C_p = R + C_v \Rightarrow C_p = \frac{7R}{2}.$$

It is possible to measure the ratio of C_p / C_v

i.e Monatomic : $\frac{C_p}{C_v} = \frac{5}{3} = 1.67$

Diatomic $= \frac{C_p}{C_v} = \frac{7}{5} = 1.4$

Polyatomic $= \frac{n+2}{n} = 1 + \frac{2}{n}$

The measure is given the symbol γ

$$\gamma = \frac{C_p}{C_v}$$

Examples of the 1st Law of Thermodynamics.

$$\delta Q = \delta U + \delta W$$

Heat supplied = Internal energy + External work.

1. Isobaric change.

$\delta W = P\delta V$

$$\delta Q = \delta U + P\delta V = C_p \delta t$$

$$\underline{\delta Q = C_v \delta t + P\delta V}$$

2. Isovolumetric change

$\delta V = \delta W = 0$

$$\underline{\delta Q = \delta U = C_v \delta t}$$

3. Isothermal change

$\delta t = 0$ $\quad \underline{\delta Q = \delta W}$

4. Adiabatic change $\quad \delta Q = 0.$

No heat travels through the walls of the container. No energy enters or leaves the gas

$$0 = \delta Q = \delta U + \delta W$$

$$dU = -\delta W$$

If a gas does work then its internal energy must fall, and so must its temperature.

During an Adiabatic change

$PV = nRT$ $\quad PV^{\gamma} = K$ (constant)

M. Bennett

Adiabatic changes.

$$PV^{\gamma} = k$$

$$TV^{\gamma-1} = \text{Constant}$$

$$P^{1-\gamma} T^{\gamma} = \text{Constant}$$

$$PV^{\gamma} = k \qquad \frac{dP}{dV} = -\gamma k^{-\gamma-1} = -\gamma PV^{-1} = \frac{-\gamma P}{V}$$

① A bicycle pump compresses 1atm to 3atm and air before valve opens + enters tyre
Air initially 290K how hot as it enters tyre?

Assuming Adiabatic change

$$T_1 P_1^{(\gamma-1)} = T_2 P_2^{(\gamma-1)}$$

$$\therefore T_2 = 290 \times \left(\frac{1}{3}\right)^{1-\gamma} = 290 \times \frac{1}{3}^{-0.4}$$

$$T_2 = 450\ K$$

② Diesel engine takes in atmosphere air + compresses it adiabatically.
Fuel then injected + ignited if hot enough. Work obtained as resulting high pressure gases force the piston back.
Ignition temp diesel = 430°C 703 K
Air enters 17°C 290K
What is the minimum compression ratio

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{Compression ratio} = \frac{V_1}{V_2} = \sqrt[\gamma-1]{\frac{T_2}{T_1}}$$

$$= 9.15$$

Compression ratio = 9.15