Physics Factsheet

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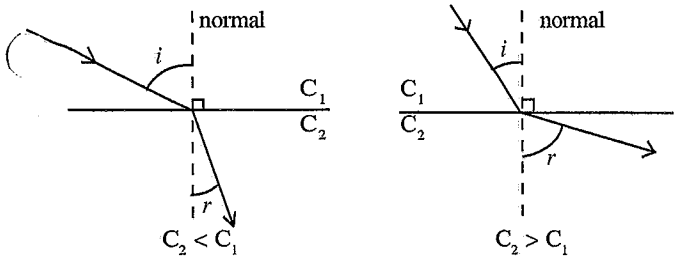
Refraction Calculations

Examiners' reports have drawn attention to weaknesses candidates have displayed in calculations concerning light rays. Calculations involving refraction caused most difficulties. This Factsheet will summarise the theory involved, but will concentrate on examples and problems.

Refraction

A light ray changes direction at a boundary between two media, if its speed changes.



(For the sake of this Factsheet, we are ignoring any reflected ray from the surface.)

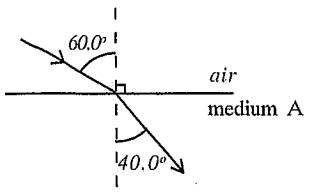
*A light ray refracts **away** from the normal when it speeds up, and **towards** the normal when it slows down.*

The equation linking angle and speed:

$$\frac{\sin i}{\sin r} = \frac{C_1}{C_2}$$

where i and r are the angles of incidence and refraction measured to the normal to the surface.

Example 1: The speed of light in air is $3.00\times10^8 ms^{-1}$. Calculate the speed of light in medium A.

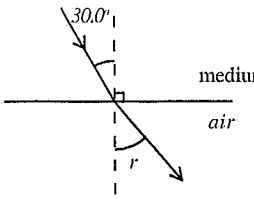


Answer:

$$\frac{\sin 60.0}{\sin 40.0} = \frac{3.00\times10^8}{C_2}$$

$$C_2 = 2.23\times10^8 ms^{-1}$$

Example 2: The speed of light in medium B is $2.40\times10^8 ms^{-1}$. Calculate the angle of refraction.



Answer:

$$\frac{\sin 30.0}{\sin r} = \frac{2.40\times10^8}{3.00\times10^8}$$

$$r = 38.7^o$$

***Exam Hint:** Check that you have not reversed the data in your calculations by matching greater speed with larger angle (measured to the normal) in your results.*

Index of refraction (refractive index)

A number is assigned to every transparent material according to the speed of light through the material. This value, n, is the refractive index of the material. n = 1.00 for air or a vacuum. For all other materials, n is greater than one.

A few examples:

Material	Refractive index, n
air / vacuum	1.00
glass	1.50
water	1.33

Different types of glass will have different refractive indices. The value of the refractive index can also depend on the wavelength of the light. We will use the values in this table for calculations.

The previous equation can now be written:

$$\frac{\sin i}{\sin r} = \frac{C_1}{C_2} = \frac{n_2}{n_1}$$

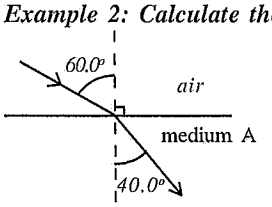
Example 1: Find the speed of light through water and glass.

Answer:

Water: $\frac{c}{3.0\times10^8} = \frac{1.00}{1.33}$ $c = 2.26\times10^8 ms^{-1}$

Glass: $\frac{c}{3.0\times10^8} = \frac{1.00}{1.50}$ $c = 2.00\times10^8 ms^{-1}$

Example 2: Calculate the refractive index of our medium A.

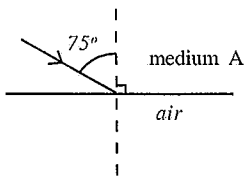


Answer:

$$\frac{n}{1.00} = \frac{\sin 60}{\sin 40}$$

$$n = 1.35$$

Example 3: A ray of light through this medium A is incident on a glass surface at an angle of 75° to the normal. Calculate the angle of refraction.

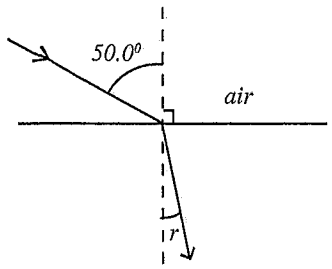


Answer: The sine of the angle of refraction works out to be greater than one. This is impossible. No refraction occurs.

When light speeds up at a boundary, and the angle of incidence is large, refraction may not occur. Total Internal Reflection occurs. See Factsheet 86 on Optical Fibres for a detailed discussion.

It is interesting to see what happens when light refracts through two layers. First of all, suppose a ray of light travels from air into glass.

Example 1: Find the angle of refraction in the situation shown.



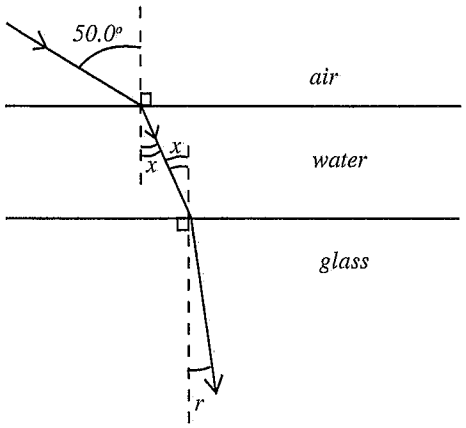
Answer:

$$\frac{\sin r}{\sin 50.0} = \frac{1.00}{1.50}$$

$$r = 30.7^0$$

Now suppose a layer of water is lying on the glass.

Example 2: Find the final angle of refraction.



Answer:

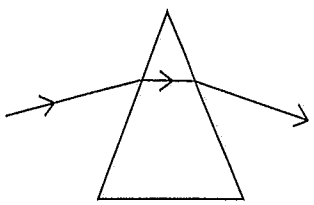
First boundary $\frac{\sin x}{\sin 50.0} = \frac{1.00}{1.33}$ $x = 35.2^o$

Second boundary $\frac{\sin r}{\sin 35.2} = \frac{1.33}{1.50}$ $r = 30.7^o$

It turns out that the intervening layer of water has no effect on the direction of the final refracted ray.

Non-parallel surfaces

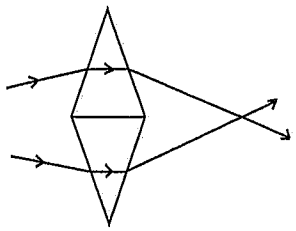
Parallel surfaces e,g, a glass block, result in the incident ray emerging parallel to its original direction. But non-parallel surfaces e.g. a triangular prism, can cause an increased deviation in the path.



(So far we have ignored the fact that different wavelengths – different colours – of light travel at different speeds through media like glass. This causes different angles of refraction, leading to a spectrum being produced from white light.)

Lenses

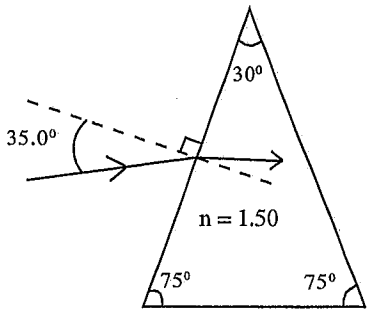
Combining two prisms shows how non-parallel surfaces can be used to focus light rays.



It is not a major step to see how curved, rather than straight, surfaces can focus all the rays from an object at one image point.

Exam-style question

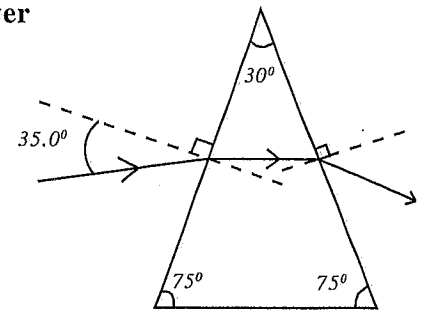
A monochromatic ray of light is incident on the surface of the triangular prism shown in the diagram.



(a) Sketch the path of the ray through the glass and emerging into the air.
(b) Calculate the angle of refraction at the first surface.
(c) Calculate the angle of incidence at the second surface.
(d) Calculate the angle of refraction at the second surface.
(e) Calculate the deviation of the emergent ray from its original direction.

Answer

(a)



(b) $r = \sin^{-1}\left(\frac{1.00}{1.50} \times \sin 35.0\right) = 22.5^0$

(c) $30 - 22.5 = 7.5^0$

(d) $r = \sin^{-1}\left(\frac{1.50}{1.00} \times \sin 7.5\right) = 11.3^o$

(e) $deviation = (35.0 - 22.5) + (11.3 - 7.5) = 16.3^0$

Practice Questions

1. An intervening layer has no effect on the direction of the final refracted ray. What effect does it have on the ray?

2. With an intervening layer, the equations could be written:

$$\frac{\sin x}{\sin y} = \frac{n_y}{n_x} \quad \text{and} \quad \frac{\sin y}{\sin z} = \frac{n_z}{n_y}$$

Show mathematically that the intervening layer should not affect the final angle z.

3. Total internal reflection can occur when light travels from our medium A (n=1.35) into air. Find the direction of the largest incident angle where refraction will occur.

4. The speed of light through air is $3.00\times10^8 ms^{-1}$, and the refractive index of air is 1.00. Use the equation to calculate the speed of light through the other materials in the table.

Material	Refractive index	Speed of light (ms^{-1})
Air	1.00	3.00×10^8
Lead fluoride	1.76	
Diamond	2.42	
Paraffin	1.44	
Ice	1.31	
Glycerol	1.47	

Answers

1. It will displace the final ray to the side i.e. parallel to the ray without the intervening layer.

2. $\frac{\sin x}{\sin y} \times \frac{\sin y}{\sin z} = \frac{n_y}{n_x} \times \frac{n_z}{n_y}$

Cancelling identical terms leads to:

$$\frac{\sin x}{\sin z} = \frac{n_z}{n_x}$$

This is the same equation that you would use if you had only the media x and z, with no intervening layer.

3. The refracted angle has a maximum value of 90° (sin r = 1.0).

$$\frac{\sin i}{\sin 90} = \frac{1.00}{1.35} \quad \text{so } i = 47.8^0$$

4.

Material	Refractive index	Speed of light (ms^{-1})
Air	1.00	3.00×10^8
Lead fluoride	1.76	**1.70×10^8**
Diamond	2.42	**1.24×10^8**
Paraffin	1.44	**2.08×10^8**
Ice	1.31	**2.29×10^8**
Glycerol	1.47	**2.04×10^8**

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